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On the zonal flow generation of electromagnetic ITG turbulence

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Brief introduction

1. High beta plasmas in many tokamak designs (MAST, STEP …)

2. Finite beta helps suppress ITG turbulence transport (Snyder 1999, *Pueschel 2010, Citrin 2014, …*).

3. Finite beta + ITG turbulence sometimes produces extreme transport (*Waltz 2010, Pueschel 2013, Rath & Peeters 2022, …*) **without linear onset of electromagnetic instabilities.**

Brief introduction

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1. High beta plasmas in many tokamak designs (MAST, STEP …)

2. Finite beta helps suppress ITG turbulence transport (Snyder 1999, *Pueschel 2010, Citrin 2014, …*).

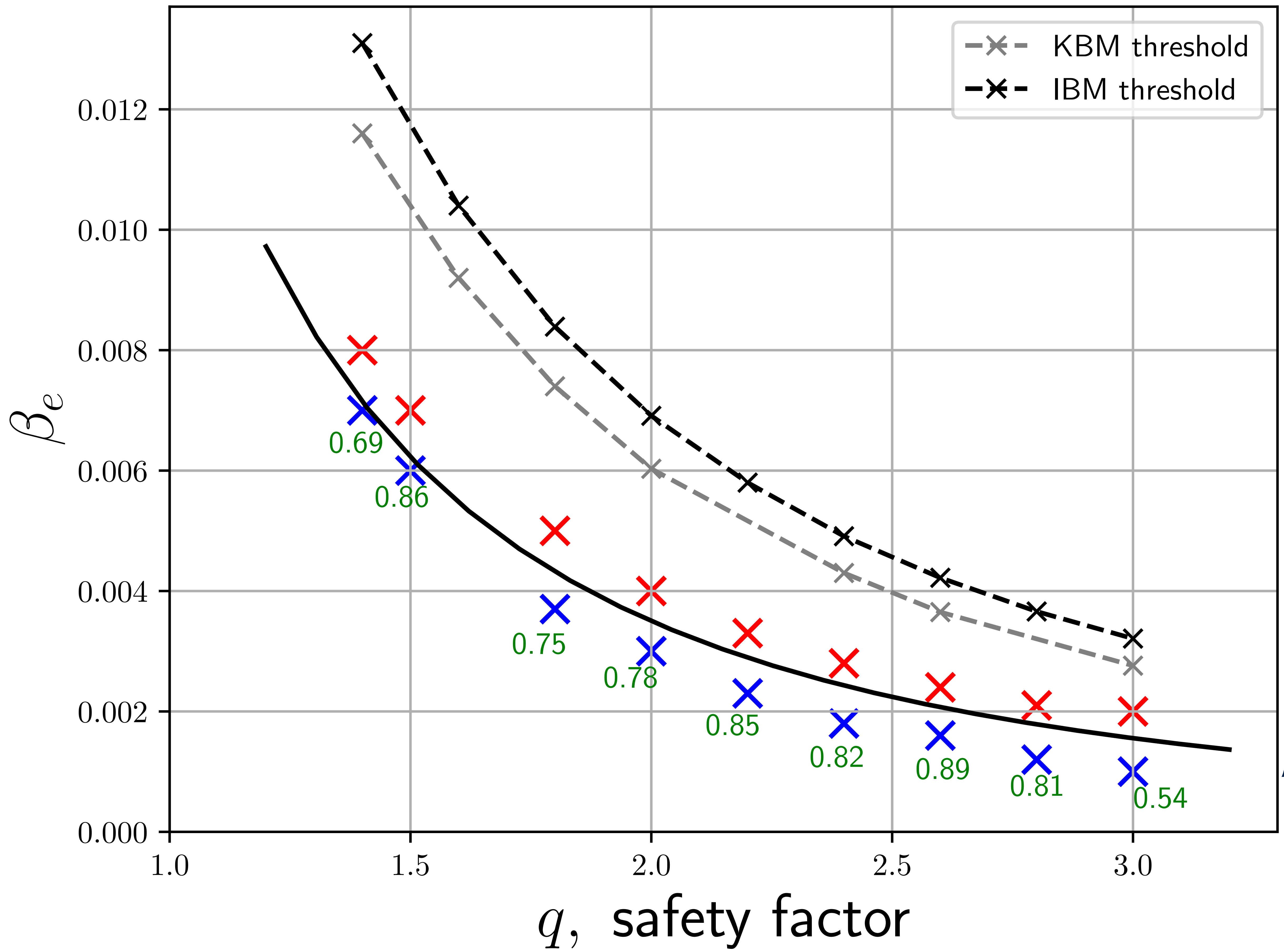
our focus

3. Finite beta + ITG turbulence sometimes produces extreme transport (*Waltz 2010, Pueschel 2013, Rath & Peeters 2022, …*) **without linear onset of electromagnetic instabilities.**

Local flux-tube, gyrokinetic simulations with *stella*

Cyclone base case (CBC)

Runaway Transition Boundary (GK CBC)



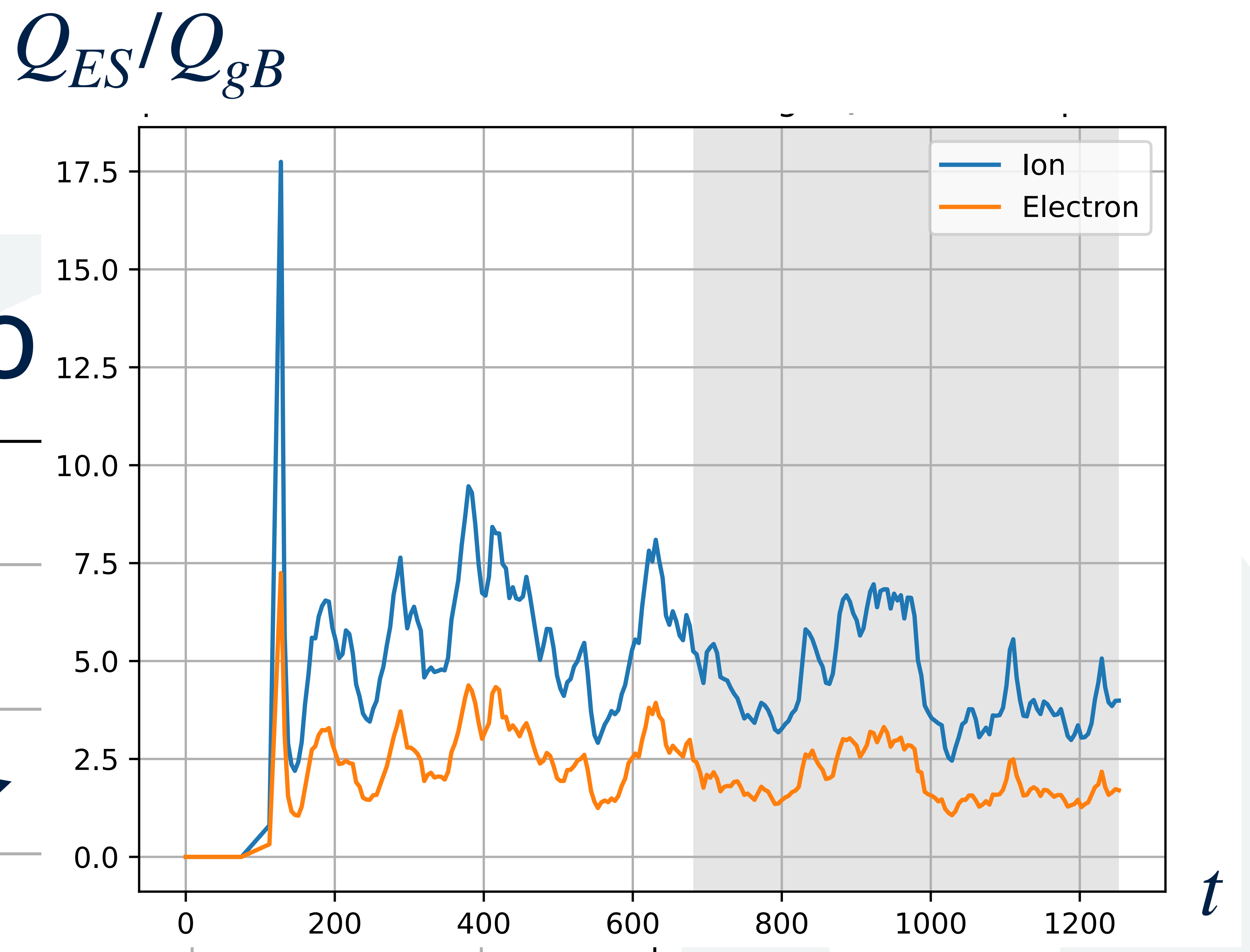
blue: converged heat flux

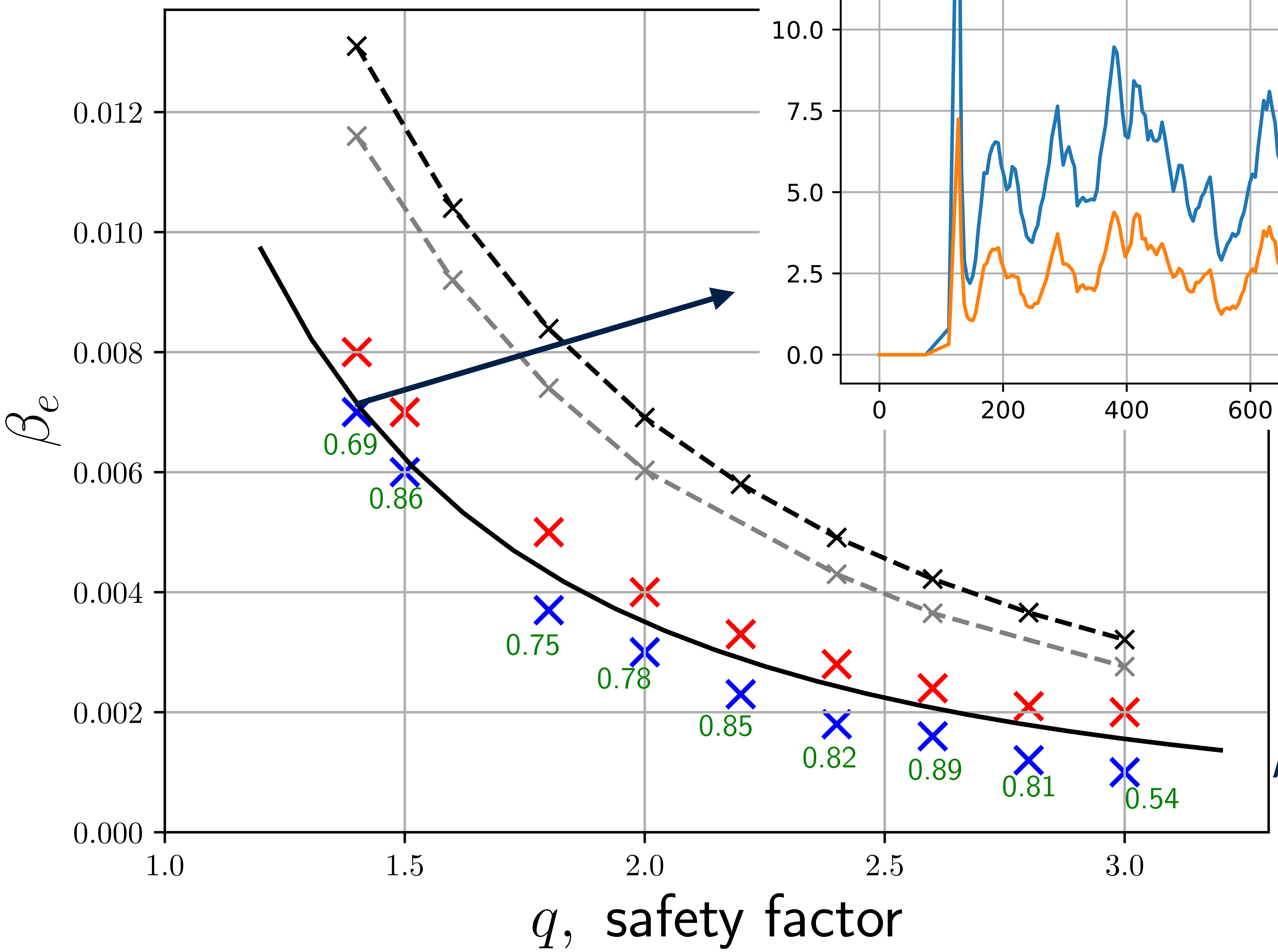
red: high flux/runaway

explain the green numbers later

$$\beta_e \propto \frac{1}{q^2}$$

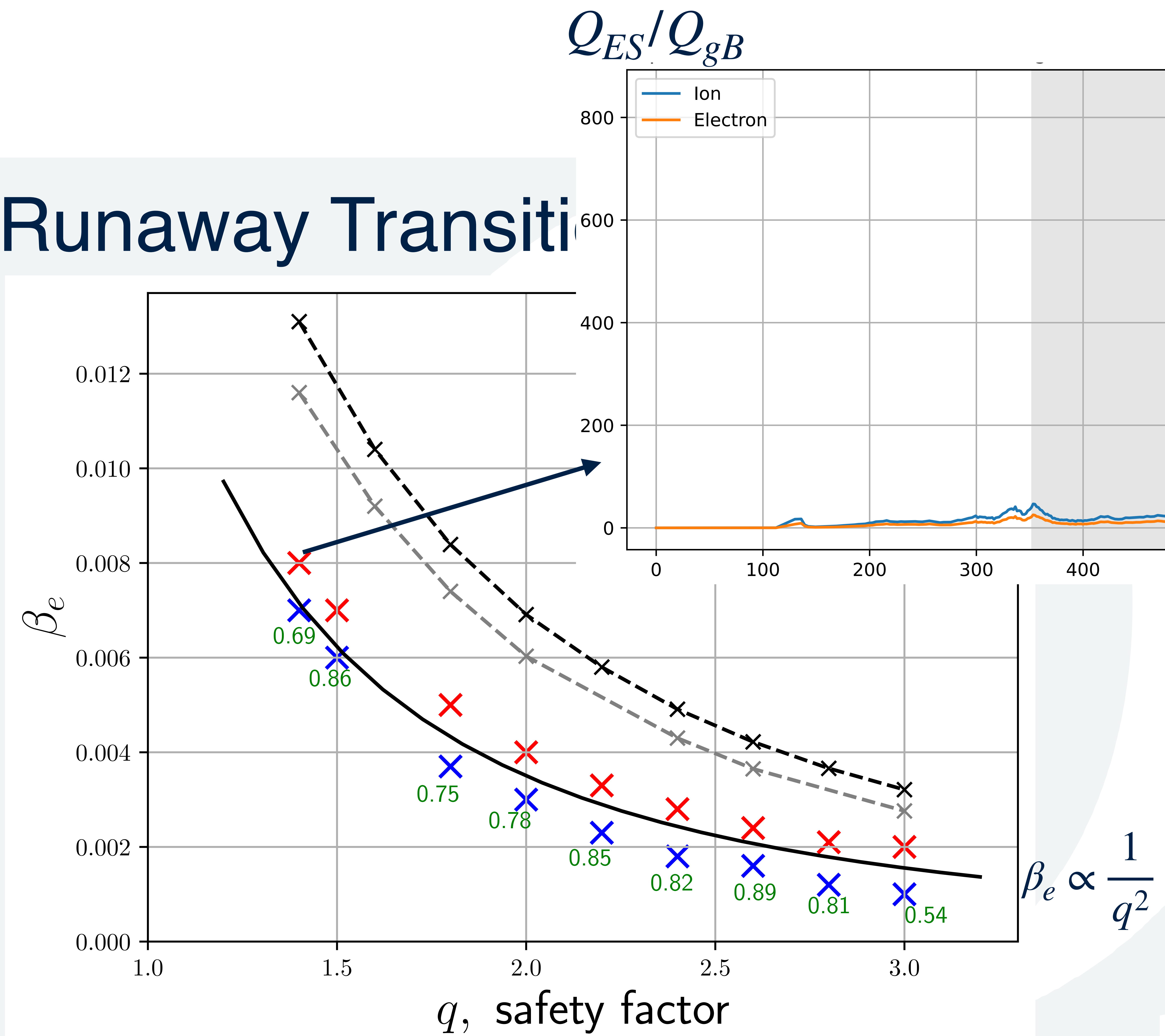
Runaway Transitio



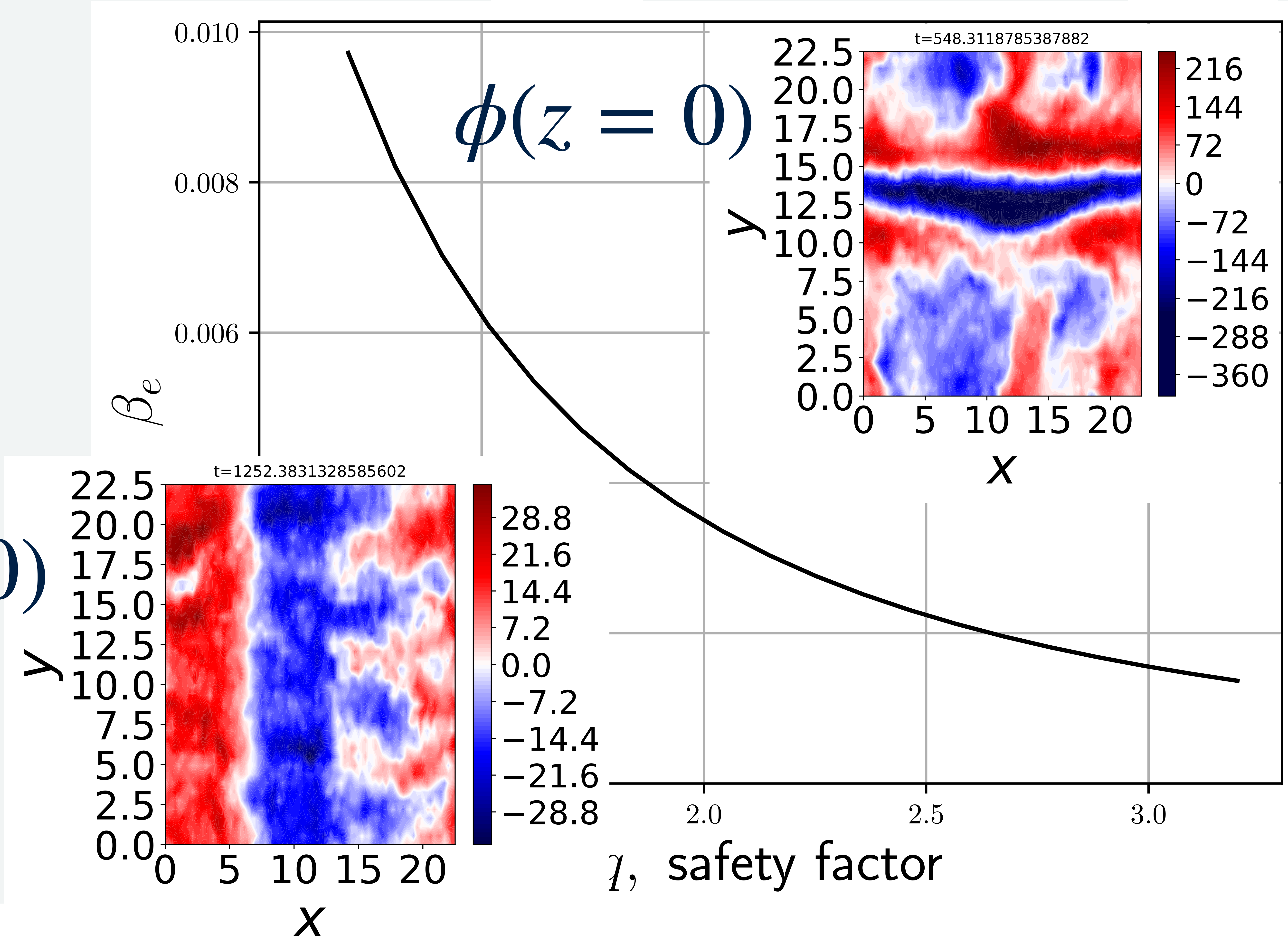


$$\beta_e \propto \frac{1}{q^2}$$

Runaway Transiti



Runaway Transition Boundary (GK CBC)



$\phi(z=0)$

$\phi(z=0)$

$$\beta_e \propto \frac{1}{q^2}$$

- For the runaway cases, the system fails to generate strong enough zonal flows.
- We will start our investigation by deriving the evolution equation of zonal flows from GK.

GK equation:

$$\frac{\partial g_s}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla z\left(\frac{\partial g_s}{\partial z} + \frac{Z_s e}{T_s}\frac{\partial\langle\phi\rangle_{\mathbf{R}_s}}{\partial z}F_s\right) + \mathbf{v}_{Ms}\cdot\left(\nabla_\perp g_s + \frac{Z_s e}{T_s}\nabla_\perp\langle\phi\rangle_{\mathbf{R}_s}F_s\right)$$

$$-\frac{\mu_s}{m_s}\mathbf{b}\cdot\nabla B\frac{\partial g_s}{\partial v_\parallel} + \langle\mathbf{v}_\chi\rangle_{\mathbf{R}_s}\cdot\nabla_\perp h_s + \langle\mathbf{v}_\chi\rangle_{\mathbf{R}_s}\cdot\nabla F_s = -\frac{Z_s e}{T_s c}F_s\frac{\partial}{\partial t}\left\langle v_\parallel A_\parallel + \mathbf{v}_\perp\cdot\mathbf{A}_\perp\right\rangle_{\mathbf{R}_s},$$

where $g_s = \langle\delta f_s\rangle_{\mathbf{R}_s}$ $\quad \delta f_s = -\frac{Z_s e\phi}{T_s}F_s + h_s$

$$\mathbf{v}_\chi = \frac{c}{B}\mathbf{b}\times\nabla_\perp\left(\phi - \frac{v_\parallel A_\parallel}{c} - \frac{\mathbf{v}_\perp\cdot\mathbf{A}_\perp}{c}\right)$$

$$\mathbf{v}_{Ms} = \frac{1}{\Omega_s}\left[\frac{1}{B}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\mathbf{b}\times\nabla B + \frac{4\pi}{B^2}v_\parallel^2\mathbf{b}\times\nabla p\right].$$

GK equation:

$$\frac{\partial g_s}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla z\left(\frac{\partial g_s}{\partial z} + \frac{Z_s e}{T_s}\frac{\partial\langle\phi\rangle_{\mathbf{R}_s}}{\partial z}F_s\right) + \mathbf{v}_{Ms}\cdot\left(\nabla_\perp g_s + \frac{Z_s e}{T_s}\nabla_\perp\langle\phi\rangle_{\mathbf{R}_s}F_s\right)$$

$$-\frac{\mu_s}{m_s}\mathbf{b}\cdot\nabla B\frac{\partial g_s}{\partial v_\parallel} + \langle\mathbf{v}_\chi\rangle_{\mathbf{R}_s}\cdot\nabla_\perp h_s + \langle\mathbf{v}_\chi\rangle_{\mathbf{R}_s}\cdot\nabla F_s = -\frac{Z_s e}{T_s c}F_s\frac{\partial}{\partial t}\langle v_\parallel A_\parallel + \mathbf{v}_\perp\cdot\mathbf{A}_\perp\rangle_{\mathbf{R}_s},$$

Quasi-neutrality:

$$\sum_s Z_s \delta n_s = \sum_s Z_s \int \mathrm{d}^3 v\left\langle g_s + \frac{Z_s e}{T_s}F_s\left(\langle\phi\rangle_{\mathbf{R}_s} - \phi\right)\right\rangle_{\mathbf{r}} = 0.$$

GK equation:

$$\frac{\partial g_s}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla z\left(\frac{\partial g_s}{\partial z} + \frac{Z_s e}{T_s}\frac{\partial\langle\phi\rangle_{\mathbf{R}_s}}{\partial z}F_s\right) + \mathbf{v}_{Ms}\cdot\left(\nabla_\perp g_s + \frac{Z_s e}{T_s}\nabla_\perp\langle\phi\rangle_{\mathbf{R}_s}F_s\right)$$

$$-\frac{\mu_s}{m_s}\mathbf{b}\cdot\nabla B\frac{\partial g_s}{\partial v_\parallel} + \langle\mathbf{v}_\chi\rangle_{\mathbf{R}_s}\cdot\nabla_\perp h_s + \langle\mathbf{v}_\chi\rangle_{\mathbf{R}_s}\cdot\nabla F_s = -\frac{Z_s e}{T_s c}F_s\frac{\partial}{\partial t}\left\langle v_\parallel A_\parallel + \mathbf{v}_\perp\cdot\mathbf{A}_\perp\right\rangle_{\mathbf{R}_s},$$

Time evolution of the electrostatic potential:

$$\frac{\partial}{\partial t}\sum_s\frac{Z_s^2 e}{T_s}\int \mathrm{d}^3v\, F_s\left(\phi - \langle\langle\phi\rangle_{\mathbf{R}_s}\rangle_{\mathbf{r}}\right) = \sum_s Z_s\int \mathrm{d}^3v\,\frac{\partial\langle g_s\rangle_{\mathbf{r}}}{\partial t}.$$

GK equation:

$$\frac{\partial g_s}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla z\left(\frac{\partial g_s}{\partial z} + \frac{Z_s e}{T_s}\frac{\partial\langle\phi\rangle_{\mathbf{R}_s}}{\partial z}F_s\right) + \mathbf{v}_{Ms}\cdot\left(\nabla_\perp g_s + \frac{Z_s e}{T_s}\nabla_\perp\langle\phi\rangle_{\mathbf{R}_s}F_s\right)$$

$$-\frac{\mu_s}{m_s}\mathbf{b}\cdot\nabla B\frac{\partial g_s}{\partial v_\parallel} + \langle\mathbf{v}_\chi\rangle_{\mathbf{R}_s}\cdot\nabla_\perp h_s + \langle\mathbf{v}_\chi\rangle_{\mathbf{R}_s}\cdot\nabla F_s = -\frac{Z_s e}{T_s c}F_s\frac{\partial}{\partial t}\langle v_\parallel A_\parallel + \mathbf{v}_\perp\cdot\mathbf{A}_\perp\rangle_{\mathbf{R}_s},$$

Time evolution of the zonal fields:

$$\frac{\partial}{\partial t}\sum_s\frac{Z_s^2 e}{T_s}\left\langle\int \mathrm{d}^3v\, F_s\left(\phi - \langle\langle\phi\rangle_{\mathbf{R}_s}\rangle_{\mathbf{r}}\right)\right\rangle_\psi = \left\langle\sum_s Z_s\int\mathrm{d}^3v\,\frac{\partial\langle g_s\rangle_{\mathbf{r}}}{\partial t}\right\rangle_\psi$$

where $\langle\cdots\rangle_\psi = \frac{1}{\partial V/\partial\psi}\int\frac{\mathrm{d}S}{|\nabla\psi|}(\cdots)$ stands for flux surface average.

Time evolution of the zonal fields:

$$\frac{\partial}{\partial t}\sum_s \frac{Z_s^2 e}{T_s}\left\langle \int \mathrm{d}^3v\, F_s\left(\phi - \left\langle\langle\phi\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}} + \frac{\left\langle\langle \mathbf{v}_\perp\cdot\mathbf{A}_\perp\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}}}{c}\right)\right\rangle_\psi = \mathit{\Pi}_{\mathrm{lin}} + \mathit{\Pi}_\phi + \mathit{\Pi}_{A_\parallel} + \mathit{\Pi}_{B_\parallel},$$

where

$$\mathit{\Pi}_{\mathrm{lin}} = -\left\langle \sum_s Z_s \int \mathrm{d}^3v\,(v_\parallel \mathbf{b}\cdot\nabla z)\left(\frac{\partial\langle g_s\rangle_{\mathbf{r}}}{\partial z} + \frac{Z_s e}{T_s}\frac{\partial\left\langle\langle\phi\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}}}{\partial z}F_s\right)\right\rangle_\psi$$
$$-\left\langle \sum_s Z_s \int \mathrm{d}^3v\,(\mathbf{v}_{Ms,x}\cdot\nabla x)\left(\frac{\partial\langle g_s\rangle_{\mathbf{r}}}{\partial x} + \frac{Z_s e}{T_s}\frac{\partial\left\langle\langle\phi\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}}}{\partial x}F_s\right)\right\rangle_\psi,$$

$$\mathit{\Pi}_\phi = -\left\langle \sum_s Z_s \int \mathrm{d}^3v\,\frac{c}{B}\left\langle\langle \mathbf{b}\times\nabla_\perp\phi\rangle_{\mathbf{R}}\cdot\nabla_\perp h_s\right\rangle_{\mathbf{r}}\right\rangle_\psi,$$

$$\mathit{\Pi}_{A_\parallel} = \left\langle \sum_s Z_s \int \mathrm{d}^3v\,\frac{v_\parallel}{B}\left\langle\langle \mathbf{b}\times\nabla_\perp A_\parallel\rangle_{\mathbf{R}}\cdot\nabla_\perp h_s\right\rangle_{\mathbf{r}}\right\rangle_\psi.$$

$$\mathit{\Pi}_{B_\parallel} = \left\langle \sum_s Z_s \int \mathrm{d}^3v\,\frac{1}{B}\left\langle\langle \mathbf{b}\times\nabla_\perp(\mathbf{v}_\perp\cdot\mathbf{A}_\perp)\rangle_{\mathbf{R}}\cdot\nabla_\perp h_s\right\rangle_{\mathbf{r}}\right\rangle_\psi.$$

Time evolution of the zonal fields:

$$\frac{\partial}{\partial t}\sum_s \frac{Z_s^2 e}{T_s}\left\langle \int \mathrm{d}^3v\, F_s\left(\phi - \left\langle\langle\phi\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}} + \frac{\left\langle\langle \mathbf{v}_\perp\cdot\mathbf{A}_\perp\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}}}{c}\right)\right\rangle_\psi = \mathit{\Pi}_{\mathrm{lin}} + \mathit{\Pi}_\phi + \mathit{\Pi}_{A_\parallel} + \mathit{\Pi}_{B_\parallel},$$

where

$$\mathit{\Pi}_{\mathrm{lin}} = -\left\langle \sum_s Z_s \int \mathrm{d}^3v\,(v_\parallel \mathbf{b}\cdot\nabla z)\left(\frac{\partial\langle g_s\rangle_{\mathbf{r}}}{\partial z} + \frac{Z_s e}{T_s}\frac{\partial\left\langle\langle\phi\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}}}{\partial z}F_s\right)\right\rangle_\psi$$
$$-\left\langle \sum_s Z_s \int \mathrm{d}^3v\,(\mathbf{v}_{Ms,x}\cdot\nabla x)\left(\frac{\partial\langle g_s\rangle_{\mathbf{r}}}{\partial x} + \frac{Z_s e}{T_s}\frac{\partial\left\langle\langle\phi\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}}}{\partial x}F_s\right)\right\rangle_\psi,$$

$$\mathit{\Pi}_\phi = -\left\langle \sum_s Z_s \int \mathrm{d}^3v\,\frac{c}{B}\left\langle\langle \mathbf{b}\times\nabla_\perp\phi\rangle_{\mathbf{R}}\cdot\nabla_\perp h_s\right\rangle_{\mathbf{r}}\right\rangle_\psi,$$

$$\mathit{\Pi}_{A_\parallel} = \left\langle \sum_s Z_s \int \mathrm{d}^3v\,\frac{v_\parallel}{B}\left\langle\langle \mathbf{b}\times\nabla_\perp A_\parallel\rangle_{\mathbf{R}}\cdot\nabla_\perp h_s\right\rangle_{\mathbf{r}}\right\rangle_\psi.$$

$$\mathit{\Pi}_{B_\parallel} = \left\langle \sum_s Z_s \int \mathrm{d}^3v\,\frac{1}{B}\left\langle\langle \mathbf{b}\times\nabla_\perp(\mathbf{v}_\perp\cdot\mathbf{A}_\perp)\rangle_{\mathbf{R}}\cdot\nabla_\perp h_s\right\rangle_{\mathbf{r}}\right\rangle_\psi.$$

Literature calls them:

Reynolds stress

Maxwell stress

Time evolution of the zonal fields:

for $\beta \ll 1$

$$\frac{\partial}{\partial t}\sum_s \frac{Z_s^2 e}{T_s}\left\langle \int \mathrm{d}^3 v\, F_s \left(\phi - \left\langle \langle \phi \rangle_{\mathbf{R}_s} \right\rangle_{\mathbf{r}} + \cancelto{0}{\frac{\left\langle \langle \mathbf{v}_\perp \cdot \mathbf{A}_\perp \rangle_{\mathbf{R}_s} \right\rangle_{\mathbf{r}}}{c}} \right) \right\rangle_\psi = \mathit{\Pi}_{\mathrm{lin}} + \mathit{\Pi}_\phi + \mathit{\Pi}_{A_\parallel} + \cancelto{0}{\mathit{\Pi}_{B_\parallel}},$$

where

$$\mathit{\Pi}_{\mathrm{lin}} = -\left\langle \sum_s Z_s \int \mathrm{d}^3 v\, (v_\parallel \mathbf{b} \cdot \nabla z) \left(\frac{\partial \langle g_s \rangle_{\mathbf{r}}}{\partial z} + \frac{Z_s e}{T_s} \frac{\partial \left\langle \langle \phi \rangle_{\mathbf{R}_s} \right\rangle_{\mathbf{r}}}{\partial z} F_s \right) \right\rangle_\psi$$
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$$\mathit{\Pi}_\phi = -\left\langle \sum_s Z_s \int \mathrm{d}^3 v\, \frac{c}{B} \left\langle \langle \mathbf{b} \times \nabla_\perp \phi \rangle_{\mathbf{R}} \cdot \nabla_\perp h_s \right\rangle_{\mathbf{r}} \right\rangle_\psi ,$$

$$\mathit{\Pi}_{A_\parallel} = \left\langle \sum_s Z_s \int \mathrm{d}^3 v\, \frac{v_\parallel}{B} \left\langle \langle \mathbf{b} \times \nabla_\perp A_\parallel \rangle_{\mathbf{R}} \cdot \nabla_\perp h_s \right\rangle_{\mathbf{r}} \right\rangle_\psi .$$

$$\mathit{\Pi}_{B_\parallel} = \left\langle \sum_s Z_s \int \mathrm{d}^3 v\, \frac{1}{B} \left\langle \langle \mathbf{b} \times \nabla_\perp (\mathbf{v}_\perp \cdot \mathbf{A}_\perp) \rangle_{\mathbf{R}} \cdot \nabla_\perp h_s \right\rangle_{\mathbf{r}} \right\rangle_\psi .$$

Time evolution of the zonal fields:

$$\frac{\partial}{\partial t}\sum_s \frac{Z_s^2 e}{T_s}\left\langle \int \mathrm{d}^3 v\, F_s\left(\phi - \left\langle\langle\phi\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}}\right)\right\rangle_\psi = \Pi_{\mathrm{lin}} + \Pi_\phi + \Pi_{A_\parallel},$$

where

$$\Pi_{\mathrm{lin}} = -\left\langle \sum_s Z_s \int \mathrm{d}^3 v\, (v_\parallel \mathbf{b}\cdot\nabla z)\left(\frac{\partial\langle g_s\rangle_{\mathbf{r}}}{\partial z} + \frac{Z_s e}{T_s}\frac{\partial\left\langle\langle\phi\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}}}{\partial z}F_s\right)\right\rangle_\psi$$
$$-\left\langle \sum_s Z_s \int \mathrm{d}^3 v\, (\mathbf{v}_{Ms,x}\cdot\nabla x)\left(\frac{\partial\langle g_s\rangle_{\mathbf{r}}}{\partial x} + \frac{Z_s e}{T_s}\frac{\partial\left\langle\langle\phi\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}}}{\partial x}F_s\right)\right\rangle_\psi,$$

$$\Pi_\phi = -\left\langle \sum_s Z_s \int \mathrm{d}^3 v\, \frac{c}{B}\left\langle\langle\mathbf{b}\times\nabla_\perp\phi\rangle_{\mathbf{R}}\cdot\nabla_\perp h_s\right\rangle_{\mathbf{r}}\right\rangle_\psi,$$

$$\Pi_{A_\parallel} = \left\langle \sum_s Z_s \int \mathrm{d}^3 v\, \frac{v_\parallel}{B}\left\langle\langle\mathbf{b}\times\nabla_\perp A_\parallel\rangle_{\mathbf{R}}\cdot\nabla_\perp h_s\right\rangle_{\mathbf{r}}\right\rangle_\psi.$$

Consider a two-species plasma — ions (Z=1) and electrons (Z = -1)

$$\frac{\partial}{\partial t} \sum_s \frac{Z_s^2 e}{T_s} \left\langle \int \mathrm{d}^3 v \, F_s \left(\phi - \left\langle \langle \phi \rangle_{\mathbf{R}_s} \right\rangle_{\mathbf{r}} \right) \right\rangle_\psi = \Pi_{\text{lin}} + \Pi_\phi + \Pi_{A_\parallel},$$

$$T_i = T_e = T$$

$$n_i = n_e = n$$

$$\left\langle \langle \phi \rangle_{\mathbf{R}_s} \right\rangle_{\mathbf{r}} \approx \phi + \frac{1}{2} \frac{v_\perp^2}{v_{\text{th}i}^2} \rho_i^2 \nabla_\perp^2 \phi, \text{ for } k_\perp \rho_i \ll 1$$

Consider a two-species plasma — ions (Z=1) and electrons (Z = -1)

$$\frac{\partial}{\partial t}\sum_s \frac{Z_s^2 e}{T_s}\left\langle \int \mathrm{d}^3 v\, F_s\Big(\phi - \big\langle \langle\phi\rangle_{\mathbf{R}_s}\big\rangle_{\mathbf{r}}\Big)\right\rangle_\psi = \Pi_{\mathrm{lin}} + \Pi_\phi + \Pi_{A_\parallel},$$

LHS

$$\approx \frac{\partial}{\partial t}\frac{e}{T}\left\langle \int \mathrm{d}^3 v F_i\Big(\phi - \big\langle \langle\phi\rangle_{\mathbf{R}_s}\big\rangle_{\mathbf{r}}\Big)\right\rangle_\psi$$

$$\approx -\frac{en}{2T}\frac{\partial}{\partial t}\langle \rho_i^2 \nabla_\perp^2 \phi\rangle_\psi$$

$$\approx -\frac{en}{2T}\frac{\partial}{\partial t}\rho_i^2 \frac{\partial^2 \langle\phi\rangle_\psi}{\partial x^2},\ \text{assuming } |\nabla x| = 1.$$

$$T_i = T_e = T$$

$$n_i = n_e = n$$

$$\big\langle \langle\phi\rangle_{\mathbf{R}_s}\big\rangle_{\mathbf{r}} \approx \phi + \frac{1}{2}\frac{v_\perp^2}{v_{\mathrm{th}i}^2}\rho_i^2 \nabla_\perp^2 \phi,\ \text{for } k_\perp \rho_i \ll 1$$

Consider a two-species plasma — ions (Z=1) and electrons (Z = -1)

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$$\frac{\partial}{\partial t}\sum_s \frac{Z_s^2 e}{T_s}\left\langle \int \mathrm{d}^3 v\, F_s\left(\phi - \langle\langle\phi\rangle_{\mathbf{R}_s}\rangle_{\mathbf{r}}\right)\right\rangle_\psi = \varPi_{\mathrm{lin}} + \varPi_\phi + \varPi_{A_\parallel},$$

LHS

$$T_i = T_e = T$$

$$n_i = n_e = n$$

$$\approx \frac{\partial}{\partial t}\frac{e}{T}\left\langle \int \mathrm{d}^3 v F_i\left(\phi - \langle\langle\phi\rangle_{\mathbf{R}_s}\rangle_{\mathbf{r}}\right)\right\rangle_\psi$$

$$\approx -\frac{en}{2T}\frac{\partial}{\partial t}\langle\rho_i^2\nabla_\perp^2\phi\rangle_\psi$$

$$\approx -\frac{en}{2T}\frac{\partial}{\partial t}\rho_i^2\frac{\partial^2\langle\phi\rangle_\psi}{\partial x^2}, \text{ assuming } |\nabla x| = 1.$$

$$\langle\langle\phi\rangle_{\mathbf{R}_s}\rangle_{\mathbf{r}} \approx \phi + \frac{1}{2}\frac{v_\perp^2}{v_{\mathrm{th}i}^2}\rho_i^2\nabla_\perp^2\phi, \text{ for } k_\perp\rho_i \ll 1$$

Zonal part of electrostatic potential

Consider a two-species plasma — ions (Z=1) and electrons (Z = -1)

$$\frac{\partial}{\partial t}\sum_s \frac{Z_s^2 e}{T_s}\left\langle \int \mathrm{d}^3v\, F_s\Big(\phi - \langle\langle\phi\rangle_{\mathbf{R}_s}\rangle_{\mathbf{r}}\Big)\right\rangle_\psi = \mathit{\Pi}_{\mathrm{lin}} + \mathit{\Pi}_\phi + \mathit{\Pi}_{A_\parallel},$$

LHS

$$\approx \frac{\partial}{\partial t}\frac{e}{T}\left\langle \int \mathrm{d}^3v F_i\Big(\phi - \langle\langle\phi\rangle_{\mathbf{R}_s}\rangle_{\mathbf{r}}\Big)\right\rangle_\psi$$

$$\approx -\frac{en}{2T}\frac{\partial}{\partial t}\langle \rho_i^2 \nabla_\perp^2\phi\rangle_\psi$$

$$\approx -\frac{en}{2T}\frac{\partial}{\partial t}\rho_i^2\frac{\partial^2\langle\phi\rangle_\psi}{\partial x^2}, \text{ assuming } |\nabla x| = 1.$$

Evolution of mean zonal energy:

$$\left\langle \langle\phi\rangle_\psi \times -\frac{en}{2T}\frac{\partial}{\partial t}\rho_i^2\frac{\partial^2\langle\phi\rangle_\psi}{\partial x^2}\right\rangle_x = \left\langle\langle\phi\rangle_\psi \mathit{\Pi}_{\mathrm{lin}}\right\rangle_x + \left\langle\langle\phi\rangle_\psi \mathit{\Pi}_\phi\right\rangle_x + \left\langle\langle\phi\rangle_\psi \mathit{\Pi}_{A_\parallel}\right\rangle_x$$

Consider a two-species plasma — ions (Z=1) and electrons (Z = -1)

$$\frac{\partial}{\partial t}\sum_s \frac{Z_s^2 e}{T_s}\left\langle \int \mathrm{d}^3 v\, F_s\left(\phi - \langle\langle\phi\rangle_{\mathbf{R}_s}\rangle_{\mathbf{r}}\right)\right\rangle_\psi = \mathit{\Pi}_{\mathrm{lin}} + \mathit{\Pi}_\phi + \mathit{\Pi}_{A_\parallel},$$

LHS

$$\approx \frac{\partial}{\partial t}\frac{e}{T}\left\langle \int \mathrm{d}^3 v F_i\left(\phi - \langle\langle\phi\rangle_{\mathbf{R}_s}\rangle_{\mathbf{r}}\right)\right\rangle_\psi$$

$$\approx -\frac{en}{2T}\frac{\partial}{\partial t}\langle \rho_i^2 \nabla_\perp^2 \phi\rangle_\psi$$

$$\approx -\frac{en}{2T}\frac{\partial}{\partial t}\rho_i^2 \frac{\partial^2 \langle\phi\rangle_\psi}{\partial x^2}, \text{ assuming } |\nabla x| = 1.$$

Evolution of mean zonal energy:

$$\left\langle \langle\phi\rangle_\psi \times -\frac{en}{2T}\frac{\partial}{\partial t}\rho_i^2 \frac{\partial^2\langle\phi\rangle_\psi}{\partial x^2}\right\rangle_x = \left\langle \langle\phi\rangle_\psi \mathit{\Pi}_{\mathrm{lin}}\right\rangle_x + \left\langle \langle\phi\rangle_\psi \mathit{\Pi}_\phi\right\rangle_x + \left\langle \langle\phi\rangle_\psi \mathit{\Pi}_{A_\parallel}\right\rangle_x$$

$$\left\langle \frac{en}{4T}\frac{\partial}{\partial t}\left(\rho_i \frac{\partial\langle\phi\rangle_\psi}{\partial x}\right)^2\right\rangle_x = \left\langle \langle\phi\rangle_\psi \mathit{\Pi}_{\mathrm{lin}}\right\rangle_x + \left\langle \langle\phi\rangle_\psi \mathit{\Pi}_\phi\right\rangle_x + \left\langle \langle\phi\rangle_\psi \mathit{\Pi}_{A_\parallel}\right\rangle_x$$

Consider a two-species plasma — ions (Z=1) and electrons (Z = -1)

$$\frac{\partial}{\partial t}\sum_s \frac{Z_s^2 e}{T_s}\left\langle \int \mathrm{d}^3 v\, F_s\left(\phi - \left\langle\langle\phi\rangle_{\mathbf{R}_s}\right\rangle_{\mathbf{r}}\right)\right\rangle_\psi = \Pi_{\mathrm{lin}} + \Pi_\phi + \Pi_{A_\parallel},$$

Evolution of mean zonal energy:

$$\left\langle \frac{en}{4T}\frac{\partial}{\partial t}\left(\rho_i \frac{\partial\langle\phi\rangle_\psi}{\partial x}\right)^2\right\rangle_x = \left\langle\langle\phi\rangle_\psi \Pi_{\mathrm{lin}}\right\rangle_x + \left\langle\langle\phi\rangle_\psi \Pi_\phi\right\rangle_x + \left\langle\langle\phi\rangle_\psi \Pi_{A_\parallel}\right\rangle_x$$

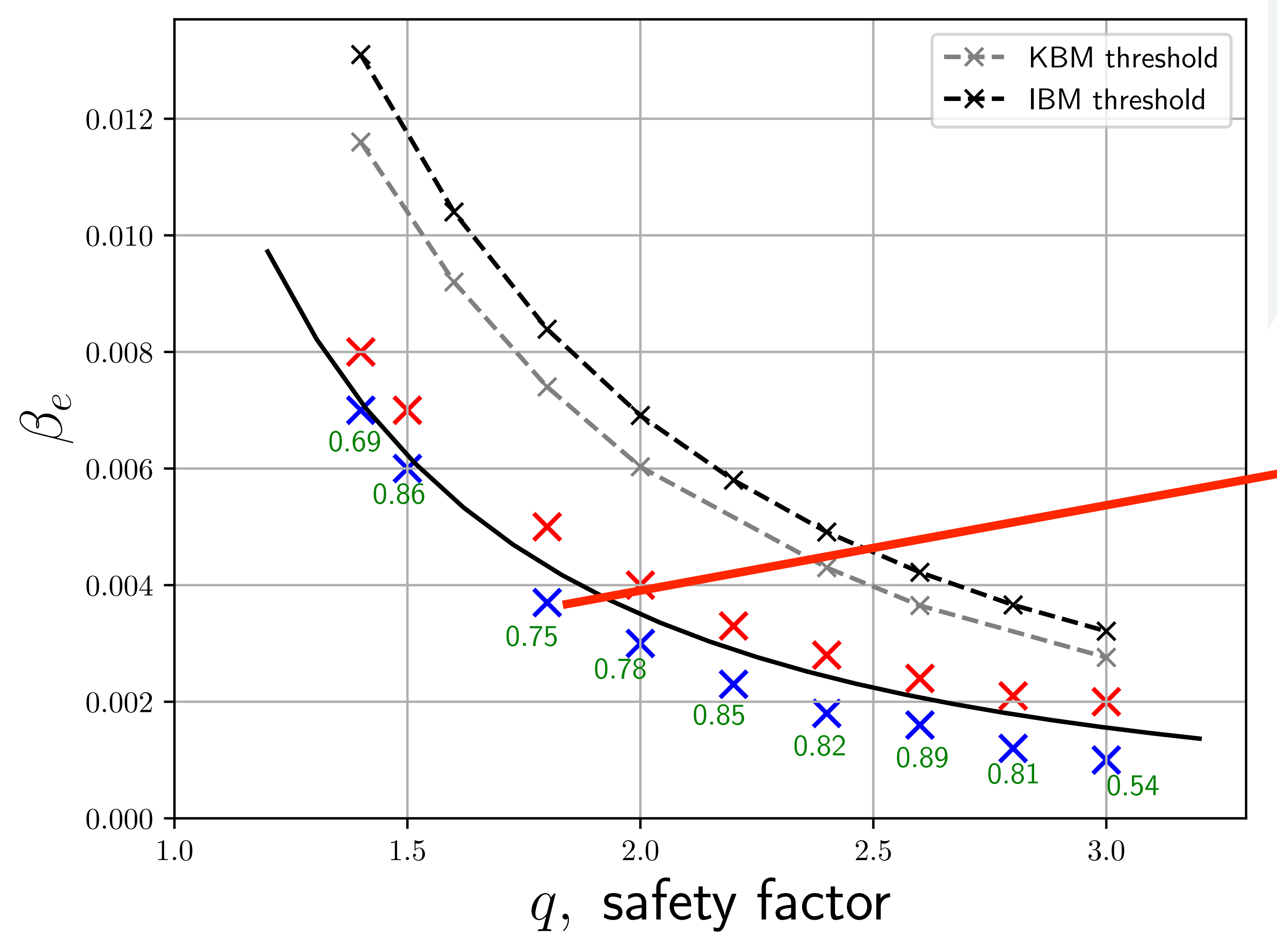
in Fourier space:

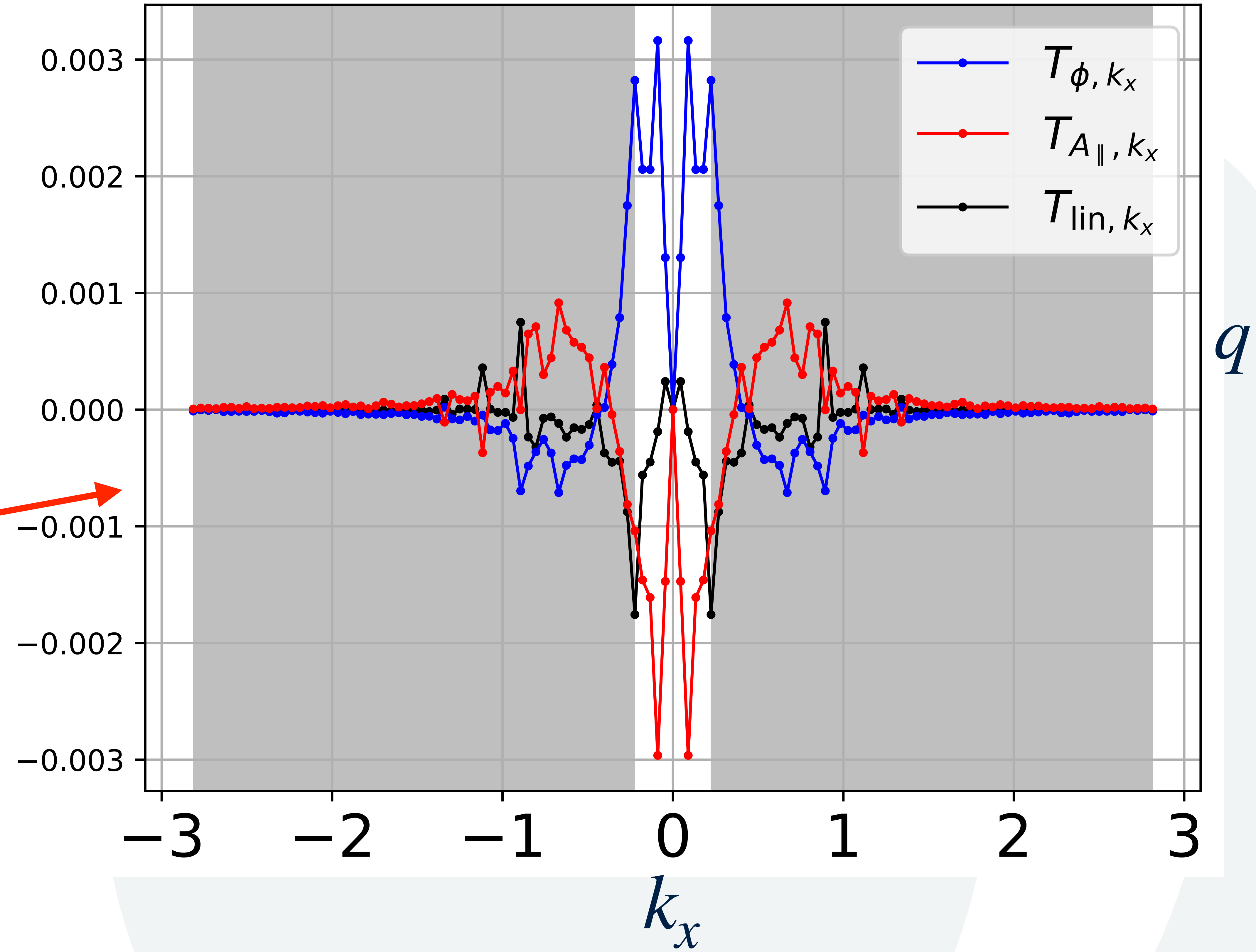
$$\begin{aligned}\sum_{k_x}\frac{en}{4T}\frac{\partial}{\partial t}k_x^2\rho_i^2|\langle\phi\rangle_{\psi,k_x}|^2 &= \sum_{k_x}\mathrm{Re}[\langle\phi\rangle^*_{\psi,k_x}\Pi_{\mathrm{lin},k_x}] + \sum_{k_x}\mathrm{Re}[\langle\phi\rangle^*_{\psi,k_x}\Pi_{\phi,k_x}] + \sum_{k_x}\mathrm{Re}[\langle\phi\rangle^*_{\psi,k_x}\Pi_{A_\parallel,k_x}]\\ &= \sum_{k_x}T_{\mathrm{lin},k_x} + \sum_{k_x}T_{\phi,k_x} + \sum_{k_x}T_{A_\parallel,k_x}\end{aligned}$$

Numerical calculation of the transfers (CBC)

Using a stella branch*: stressdiag*

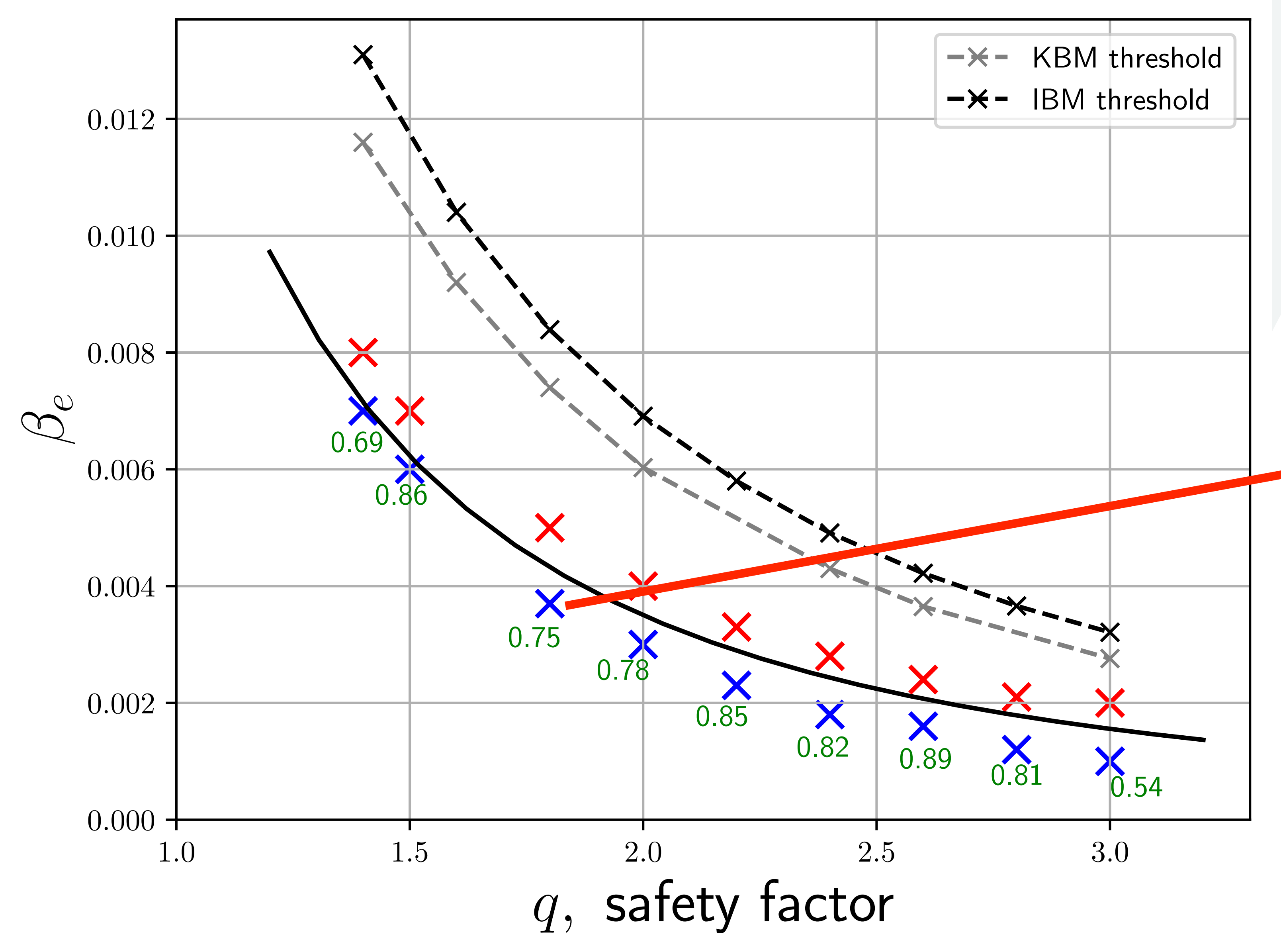
Time averaged transfer spectrum

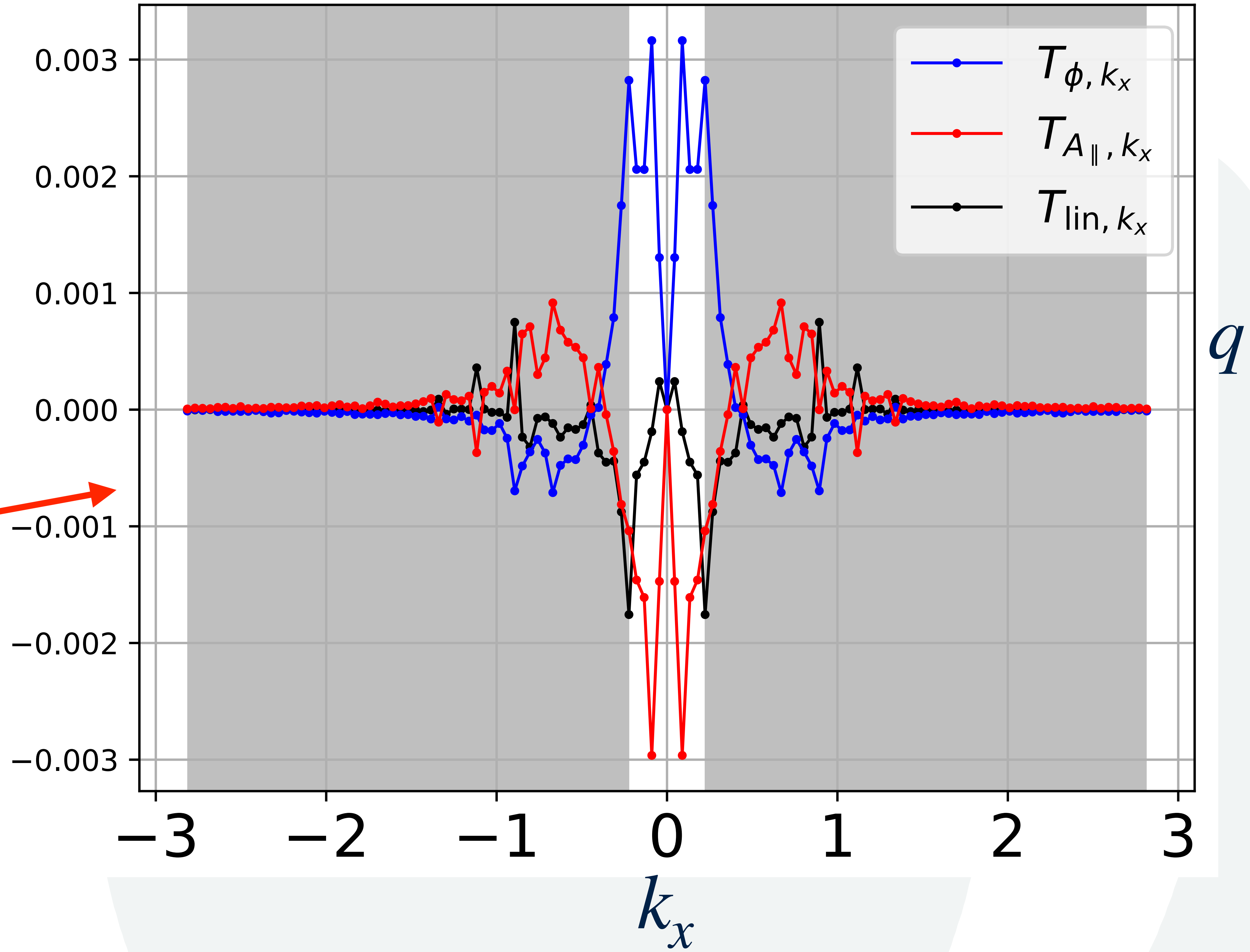




$q = 1.8,\ \beta_e = 0.0037$

Time averaged transfer spectrum





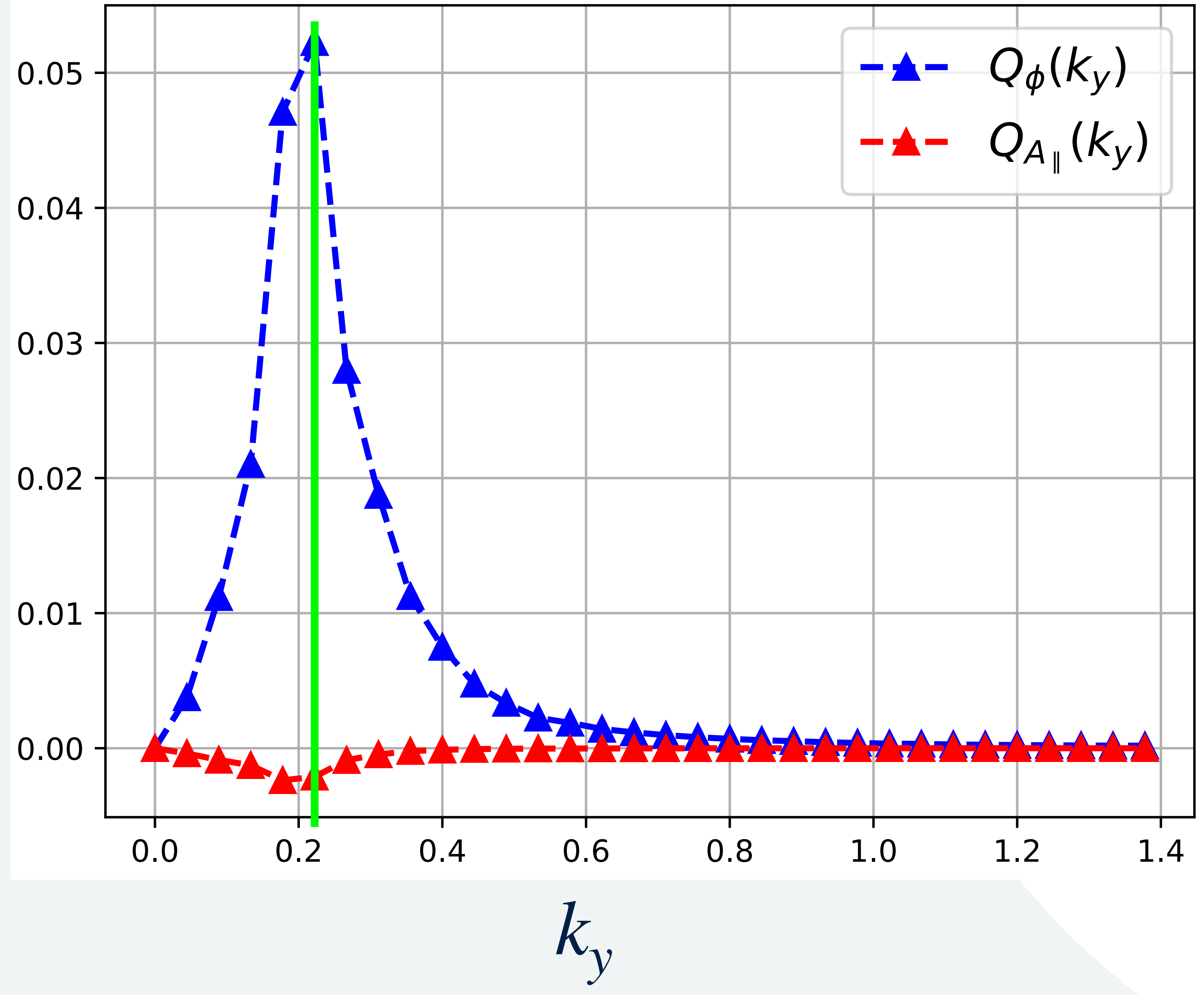
$q = 1.8,\ \beta_e = 0.0037$

$$\sum_{k_x} \frac{en}{4T}\frac{\partial}{\partial t} k_x^2 \rho_i^2 |\langle\phi\rangle_{\psi,k_x}|^2 = \sum_{k_x} \mathrm{Re}[\langle\phi\rangle^*_{\psi,k_x} \Pi_{\mathrm{lin},k_x}] + \sum_{k_x} \mathrm{Re}[\langle\phi\rangle^*_{\psi,k_x} \Pi_{\phi,k_x}] + \sum_{k_x} \mathrm{Re}[\langle\phi\rangle^*_{\psi,k_x} \Pi_{A_\parallel,k_x}]$$

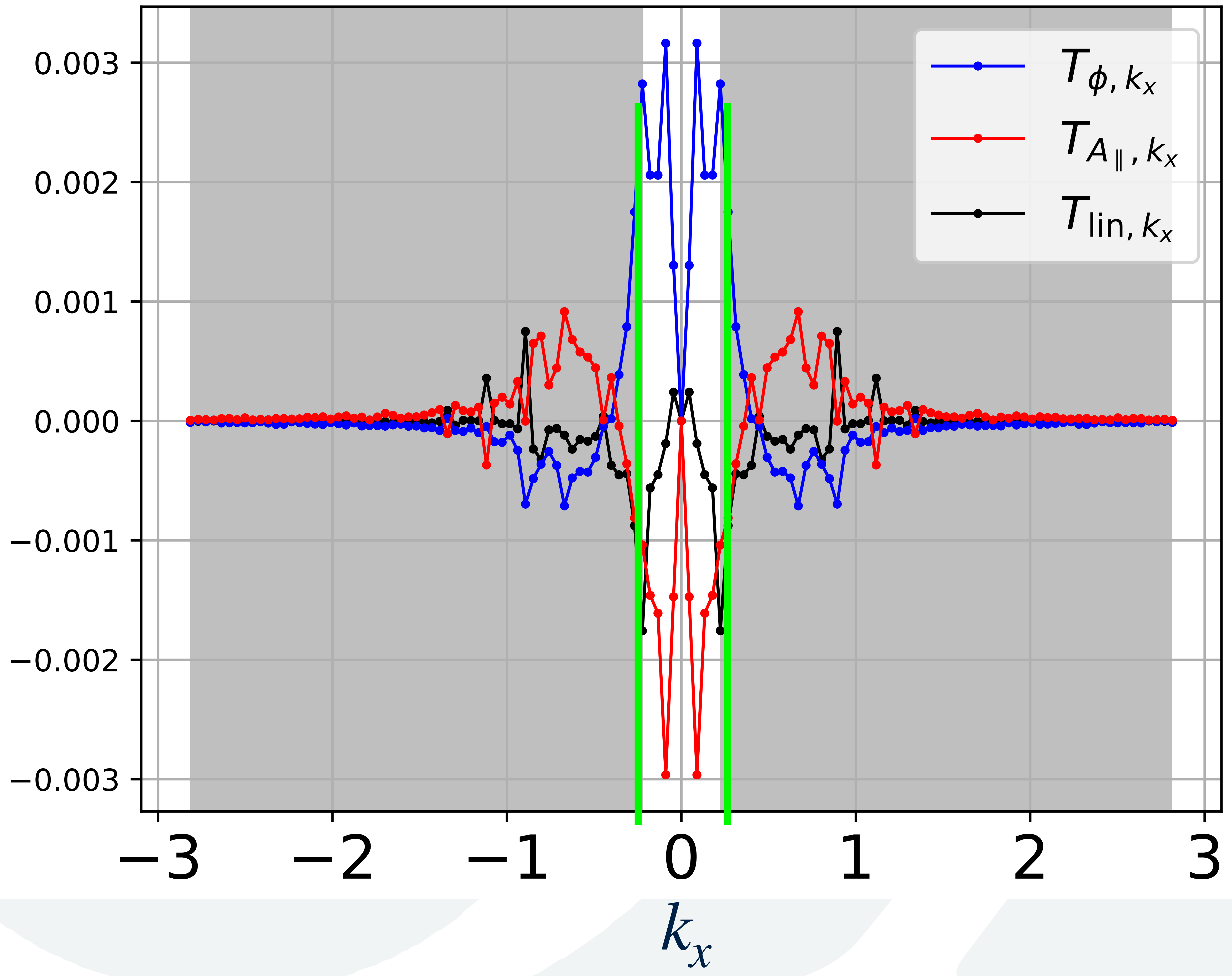
$$= \sum_{k_x} T_{\mathrm{lin},k_x} + \sum_{k_x} T_{\phi,k_x} + \sum_{k_x} T_{A_\parallel,k_x}$$

$q = 1.8,\ \beta_e = 0.0037$

Heat flux spectrum



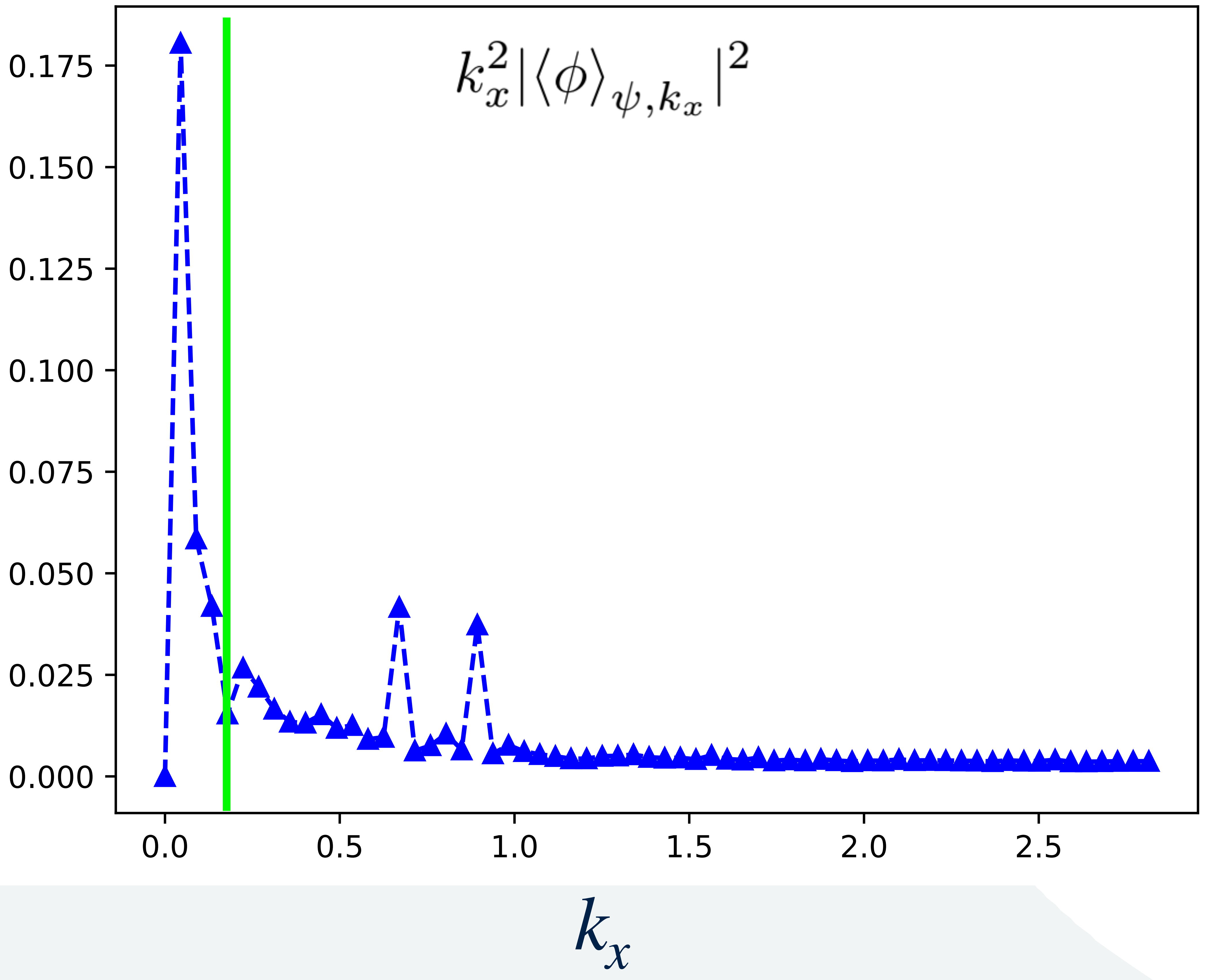
Time averaged transfer spectrum



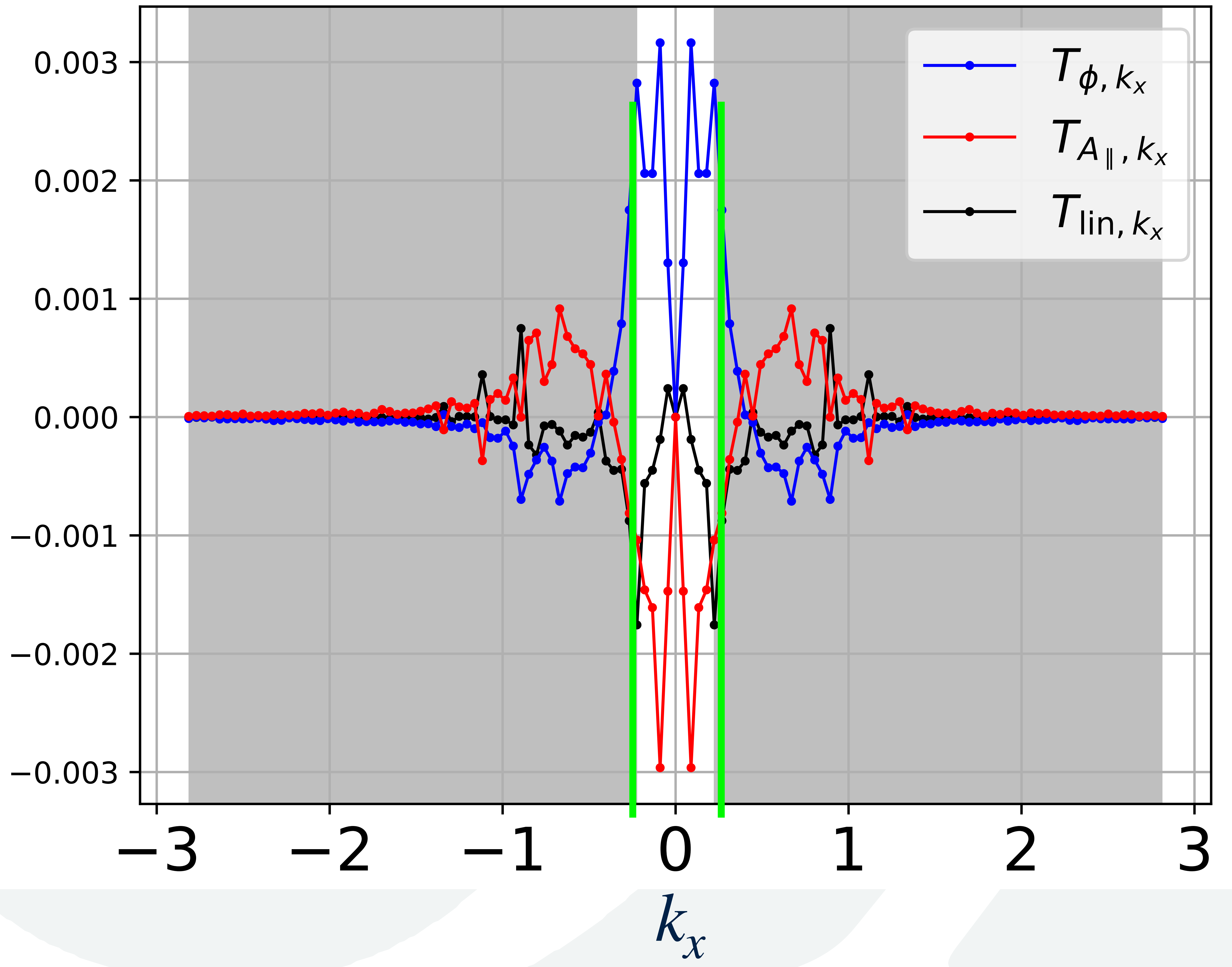
Only the large scale contributions to the transfers are considered

$q = 1.8, \beta_e = 0.0037$

Zonal flow spectrum



Time averaged transfer spectrum



Only the large scale contributions to the transfers are considered

$q = 1.8, \beta_e = 0.0037$

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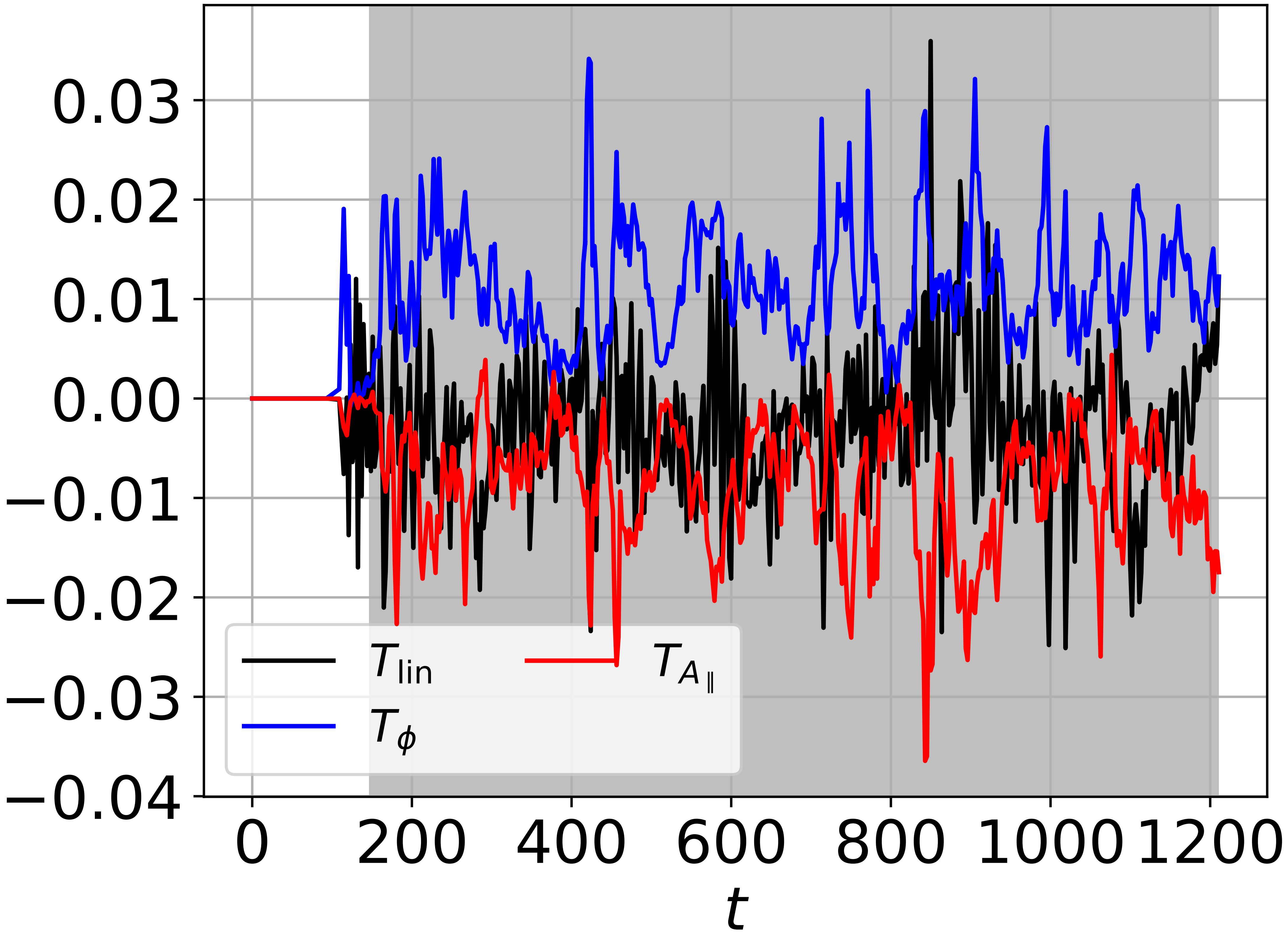
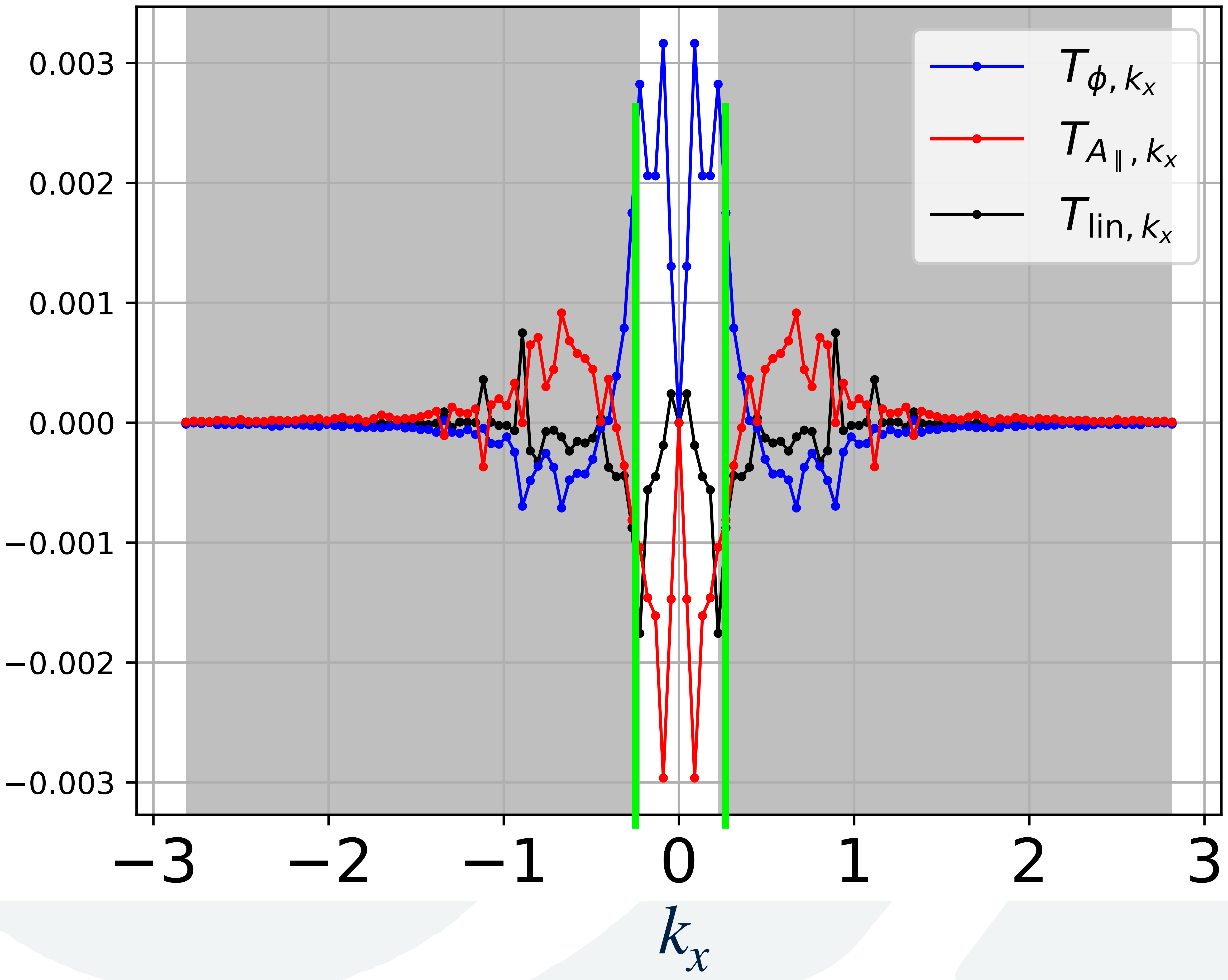
Large scale transfers

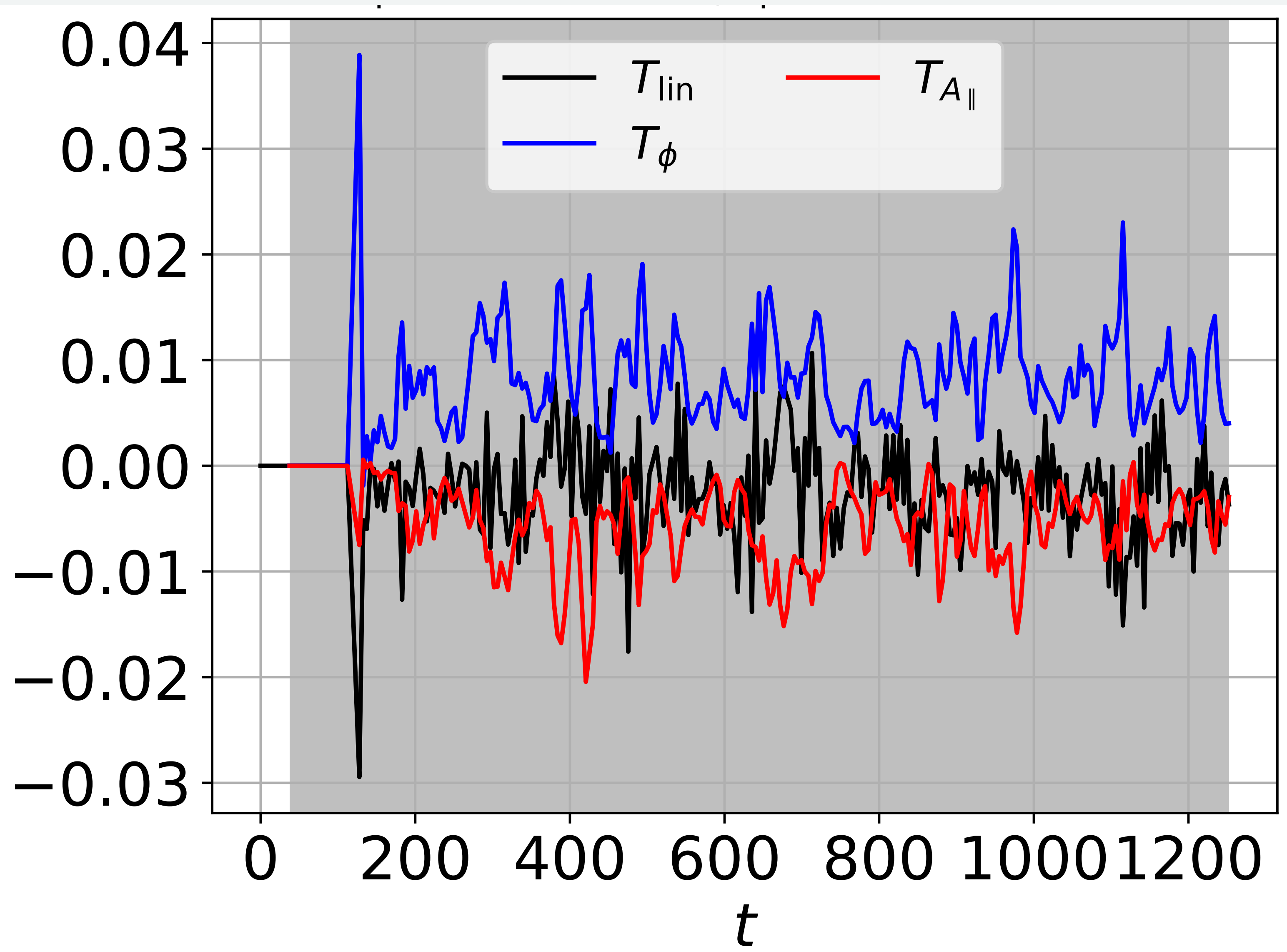


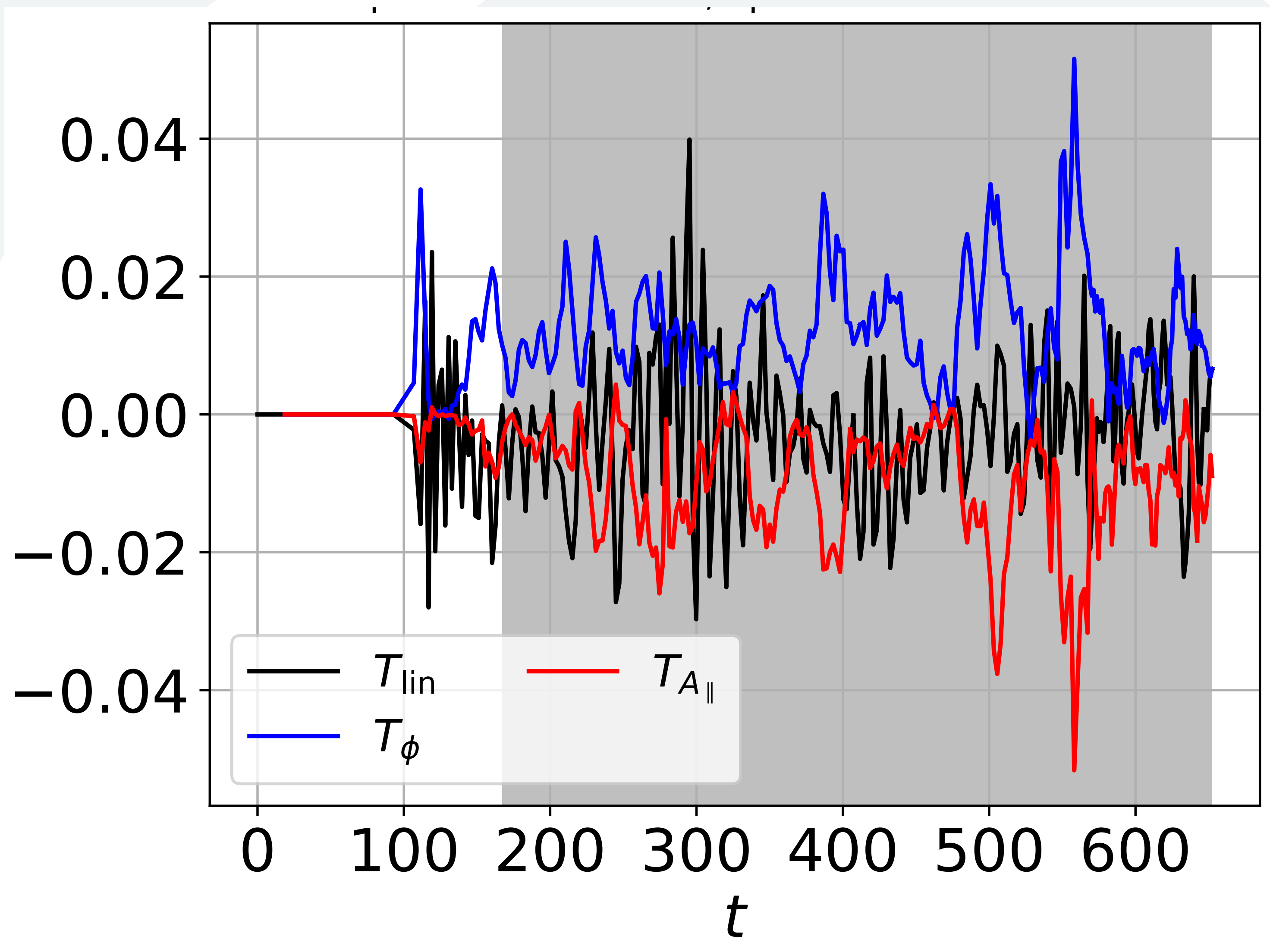
Time averaged transfer spectrum

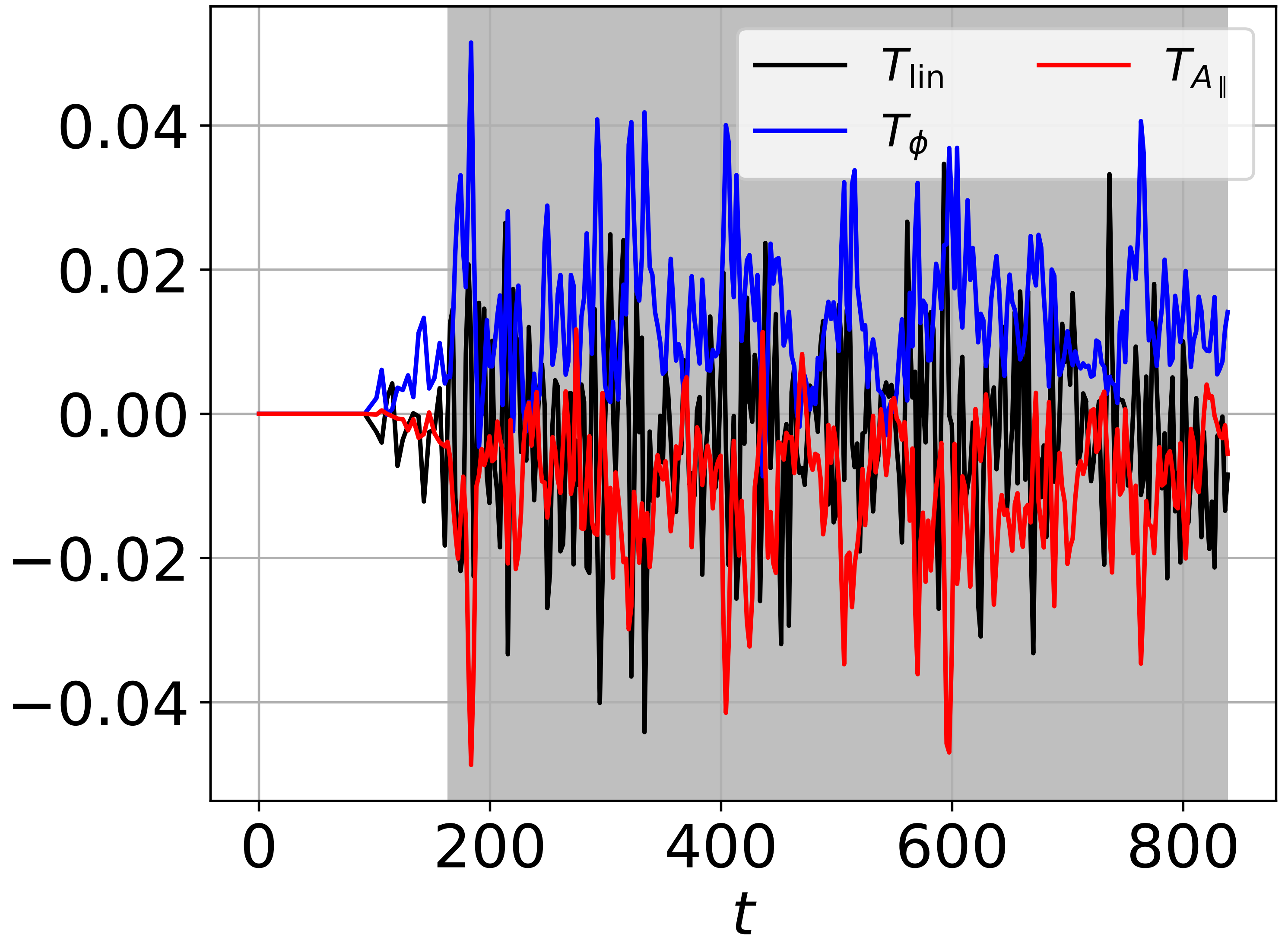


Other marginal cases

Large scale transfers





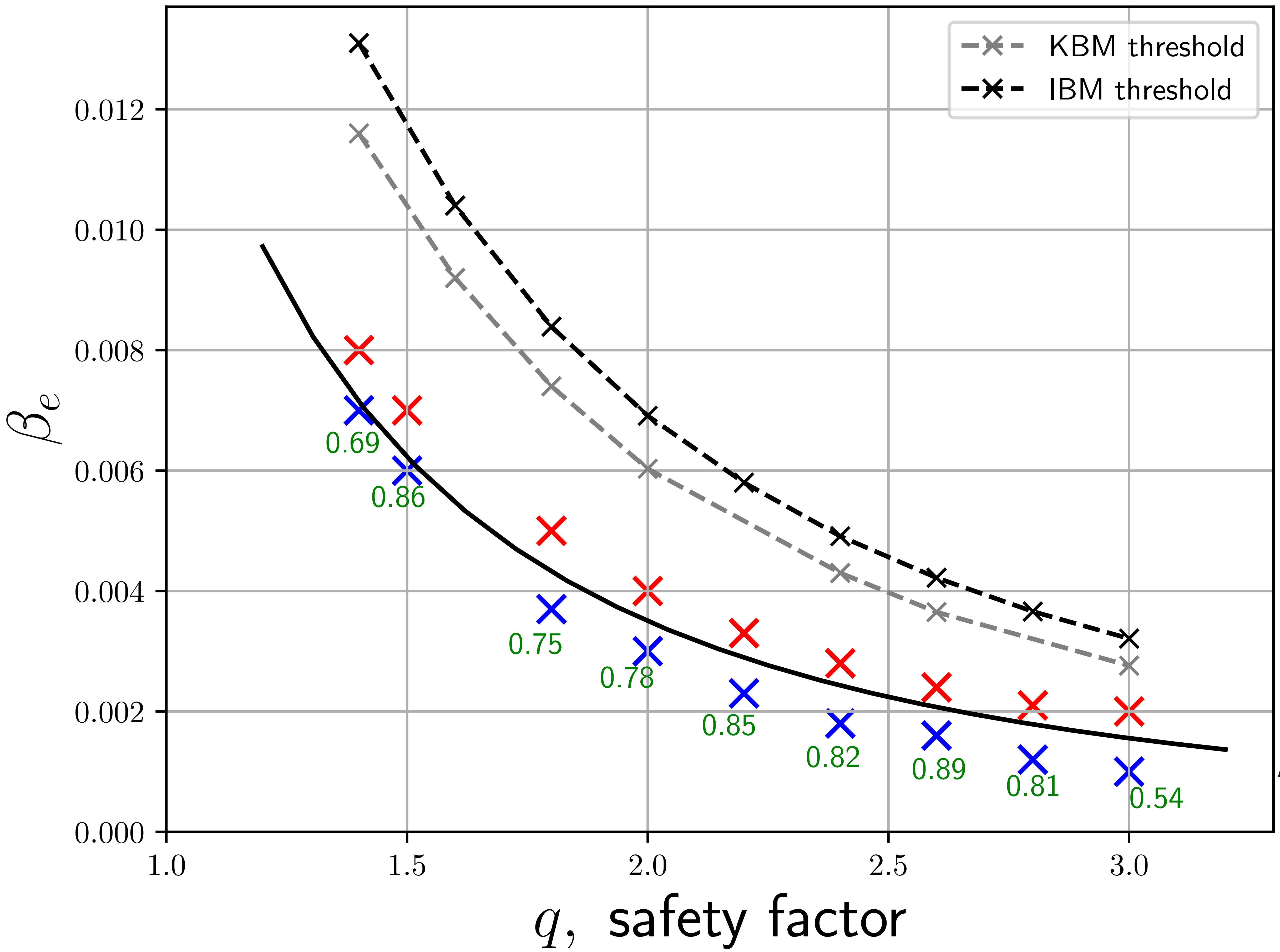


$q = 1.4,\ \beta_e = 0.007$

$q = 2.0,\ \beta_e = 0.003$

$q = 2.8,\ \beta_e = 0.0012$

Runaway Transition Boundary (GK CBC)



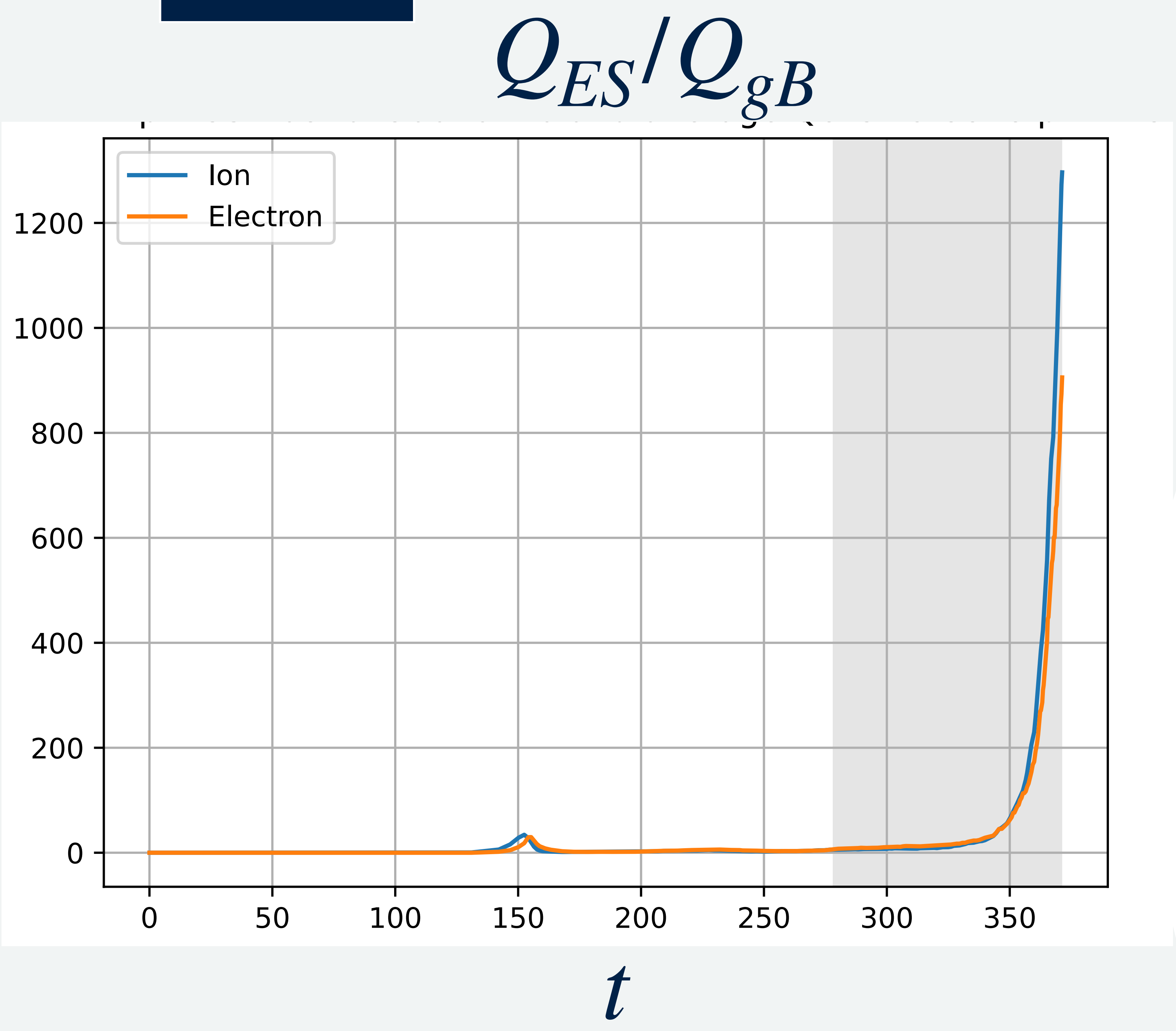
$$\beta_e \propto \frac{1}{q^2}$$

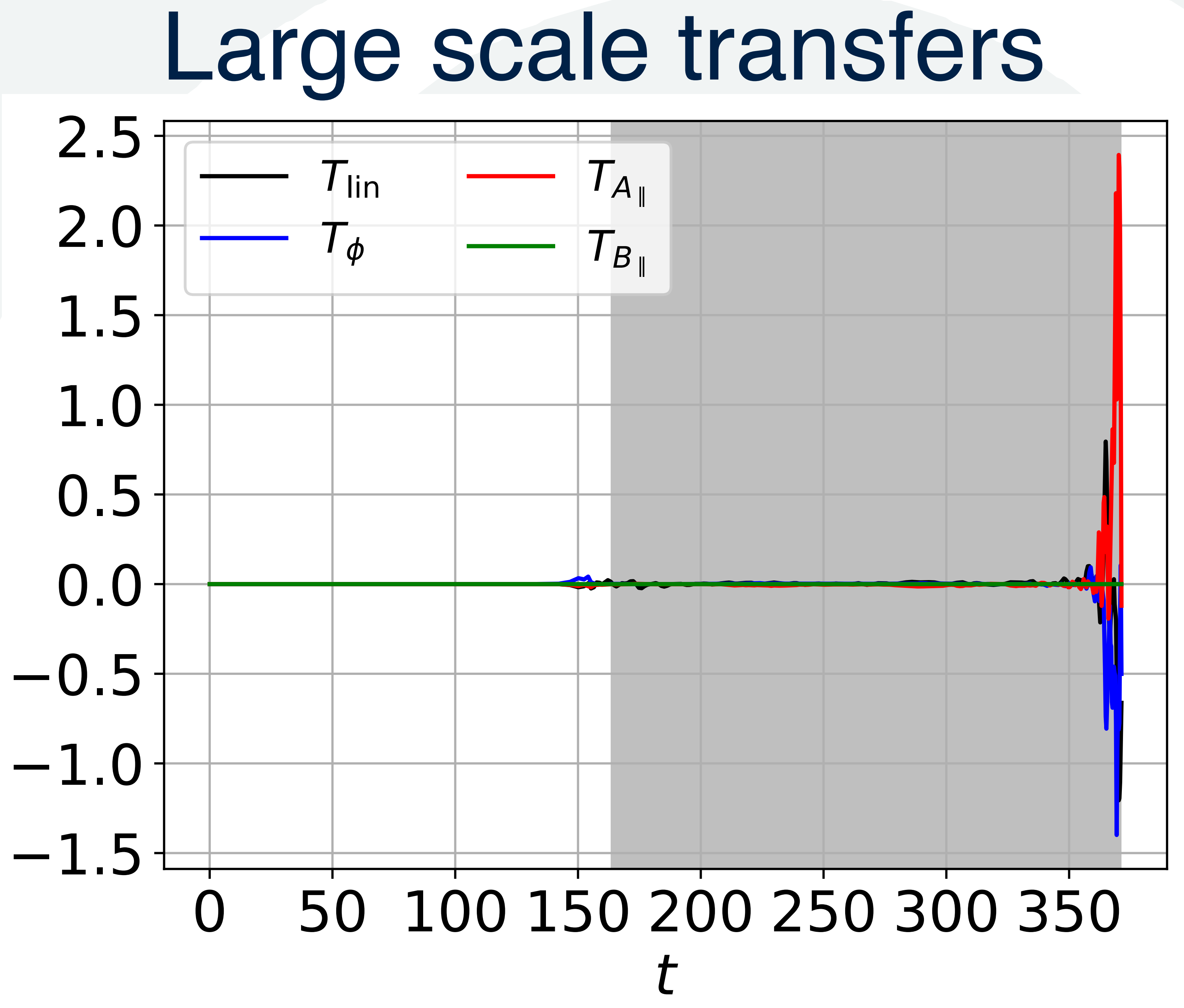
blue: converged heat flux

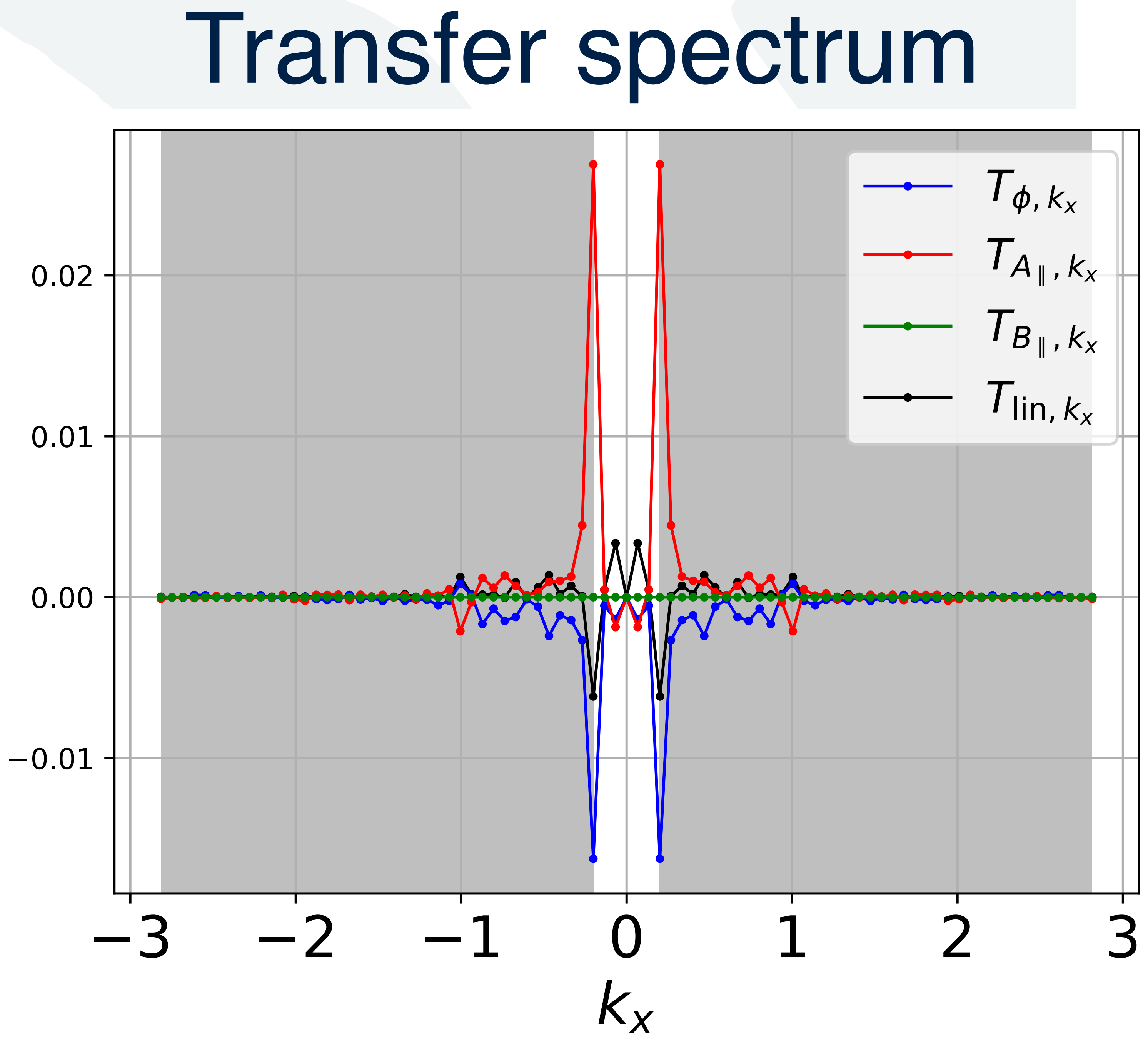
red: high flux/runaway

Green numbers: $\left|\frac{T_{A_\parallel}}{T_\phi}\right|$

A runaway case: $q = 1.4,\ \beta_e = 0.01$

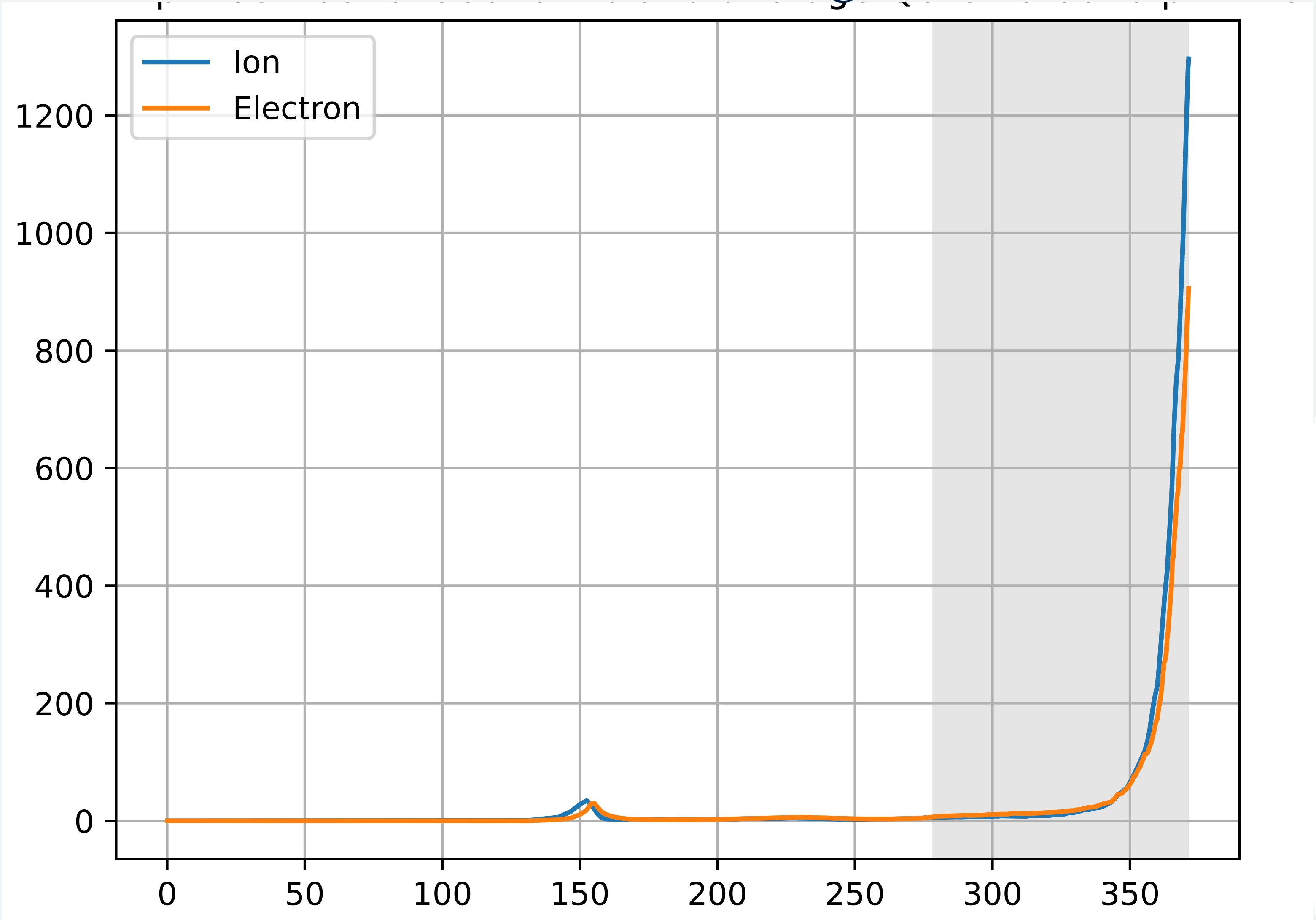






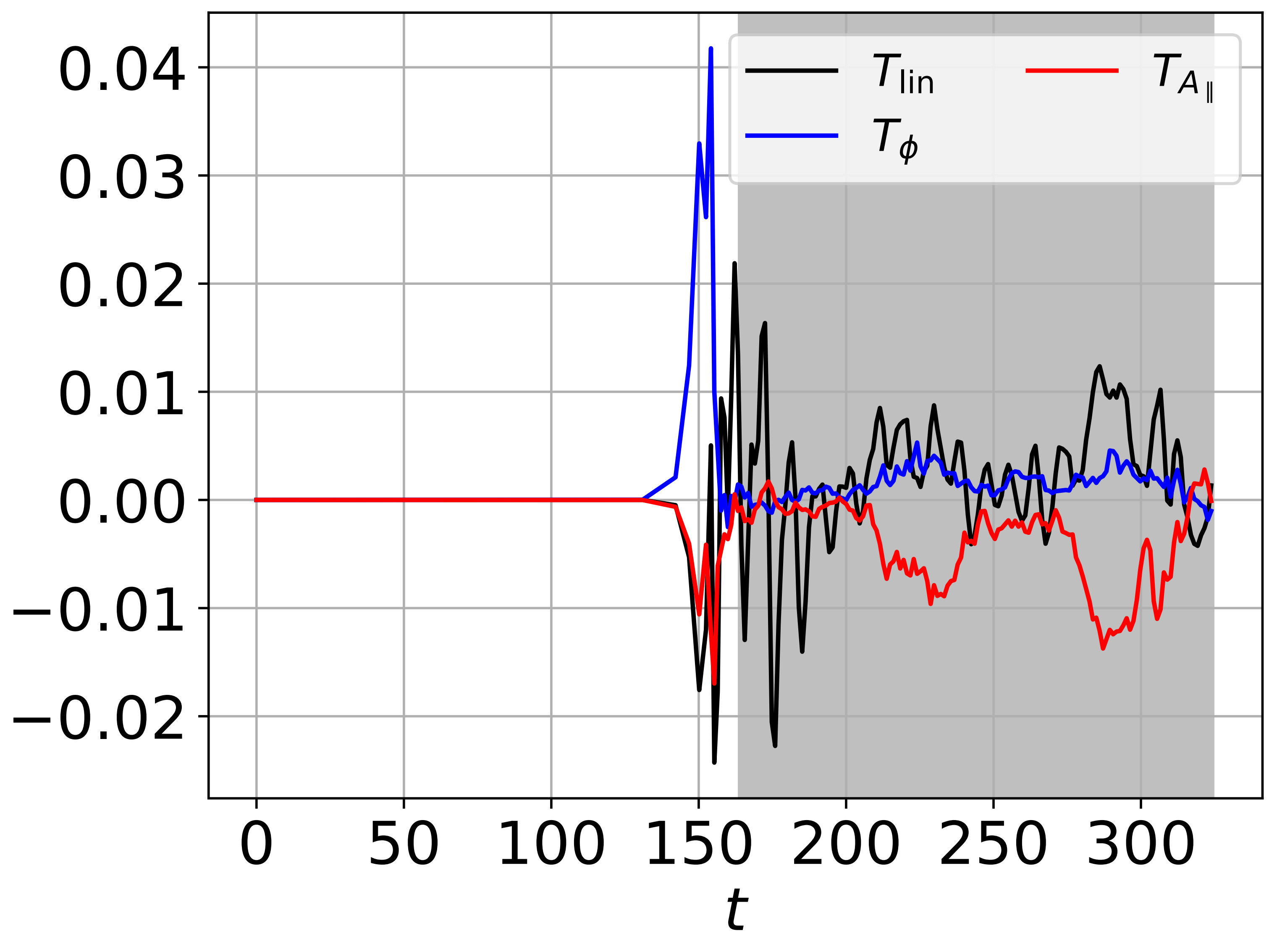
A runaway case: $q = 1.4, \beta_e = 0.01$

Q_{ES}/Q_{gB}

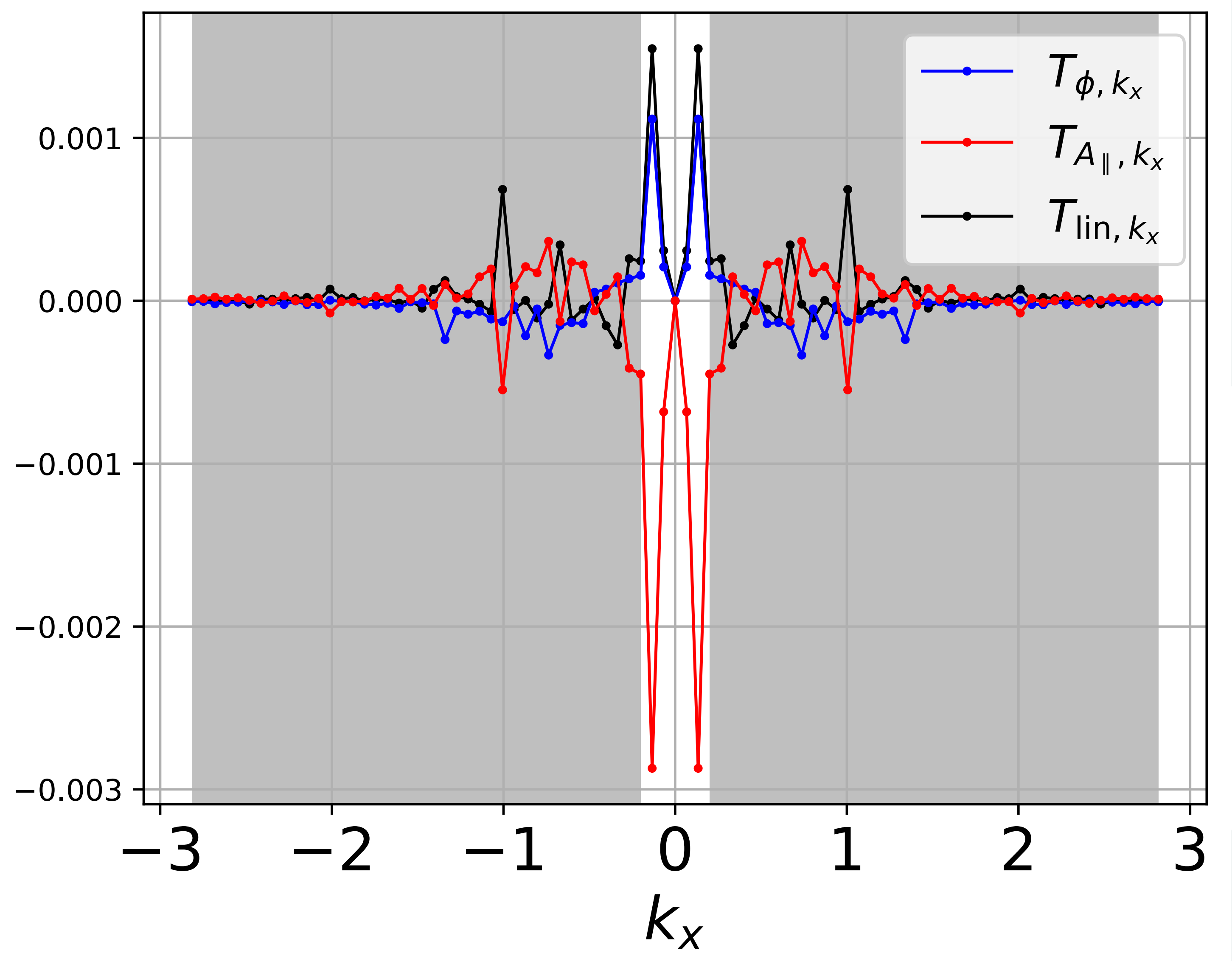


t

Large scale transfers



Transfer spectrum



A runaway case: $q = 1.4, \beta_e = 0.01$ due to nonlinear excitation of dangerous electromagnetic modes? Likely no.

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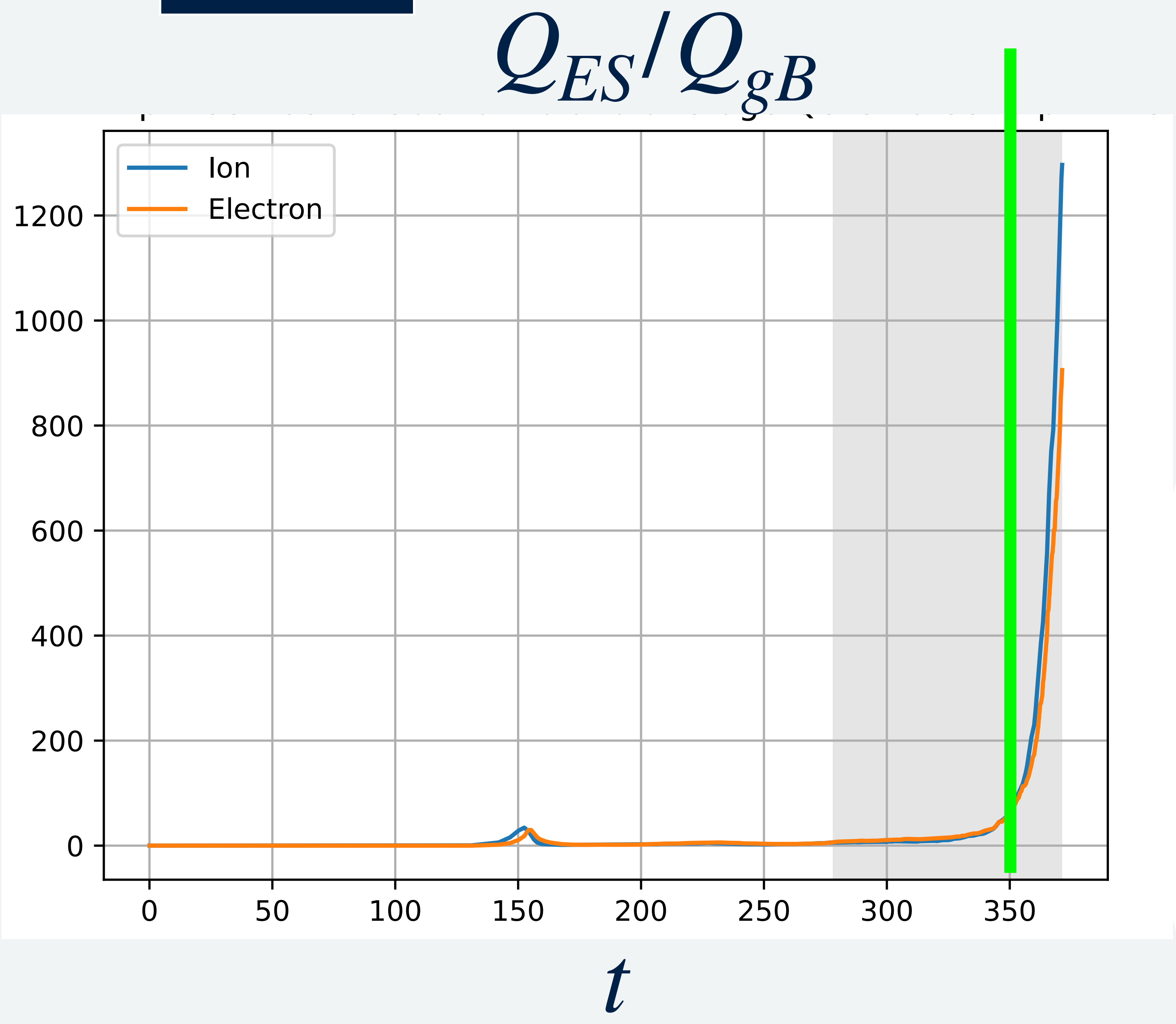

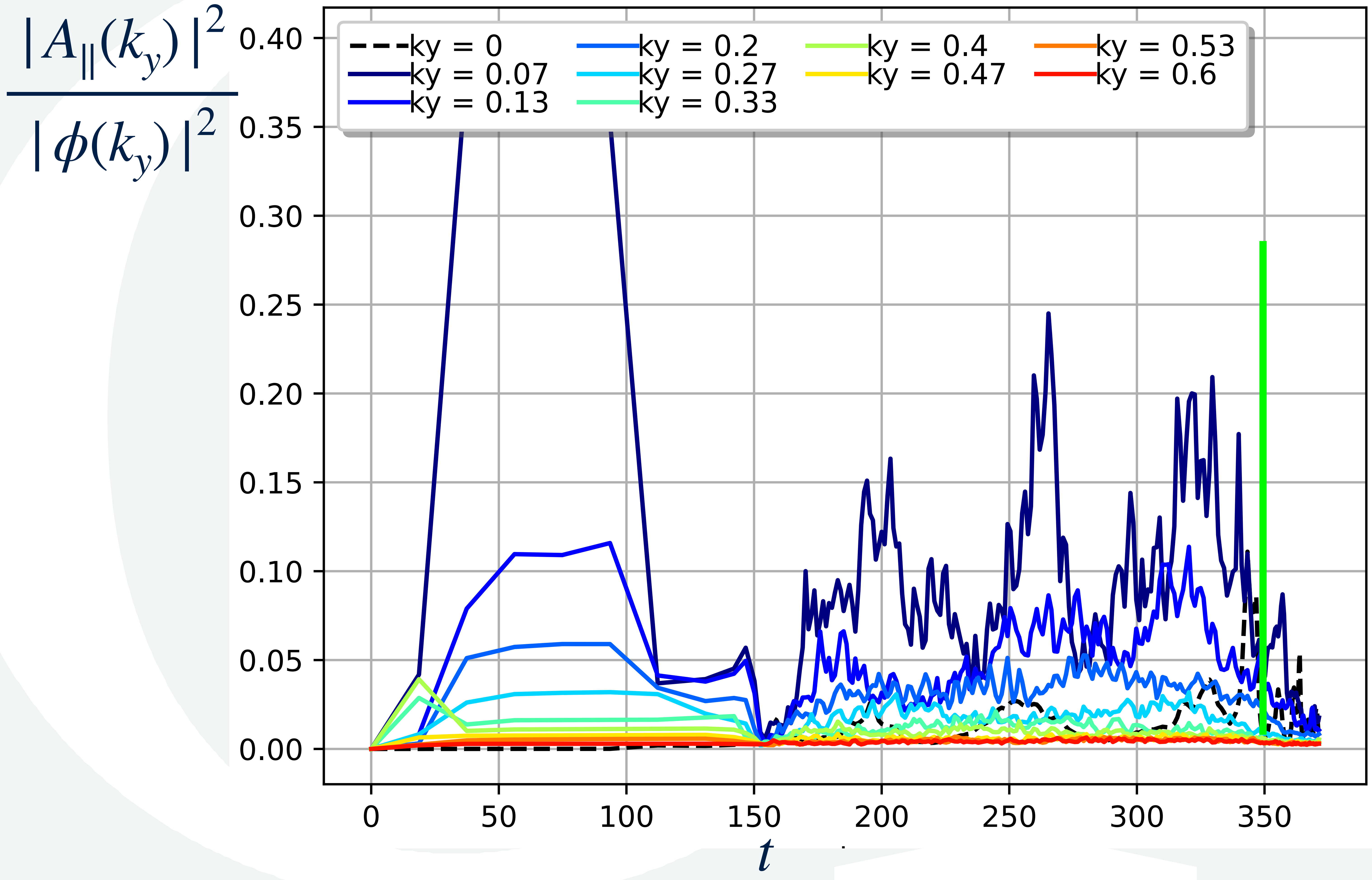

Only first 10 k_y modes are shown

Further evidence (ST40)

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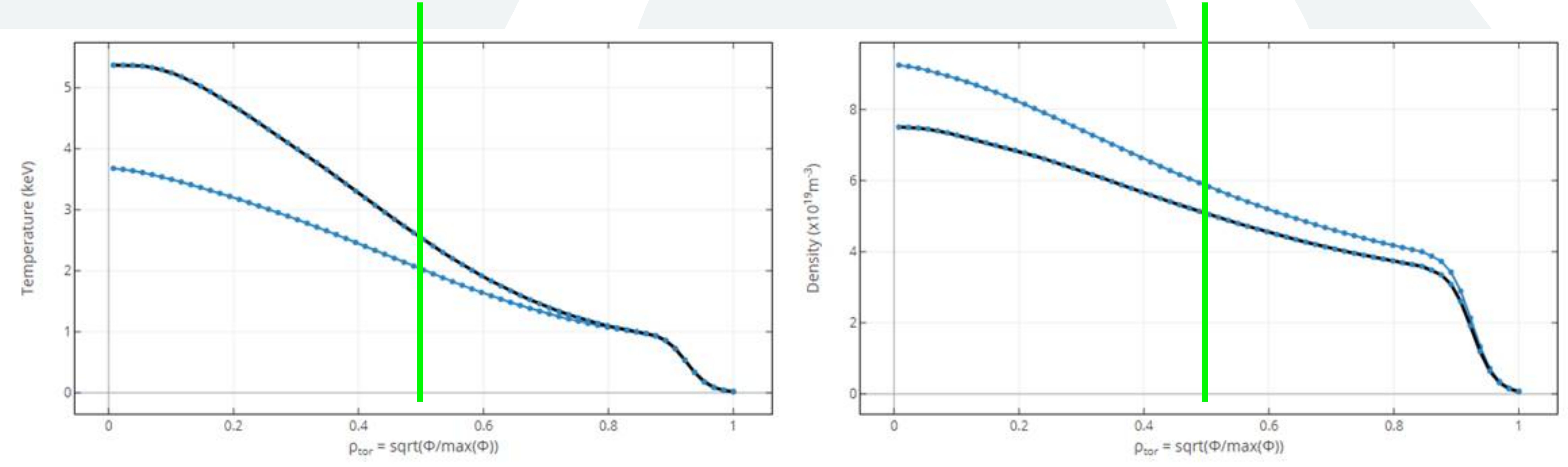
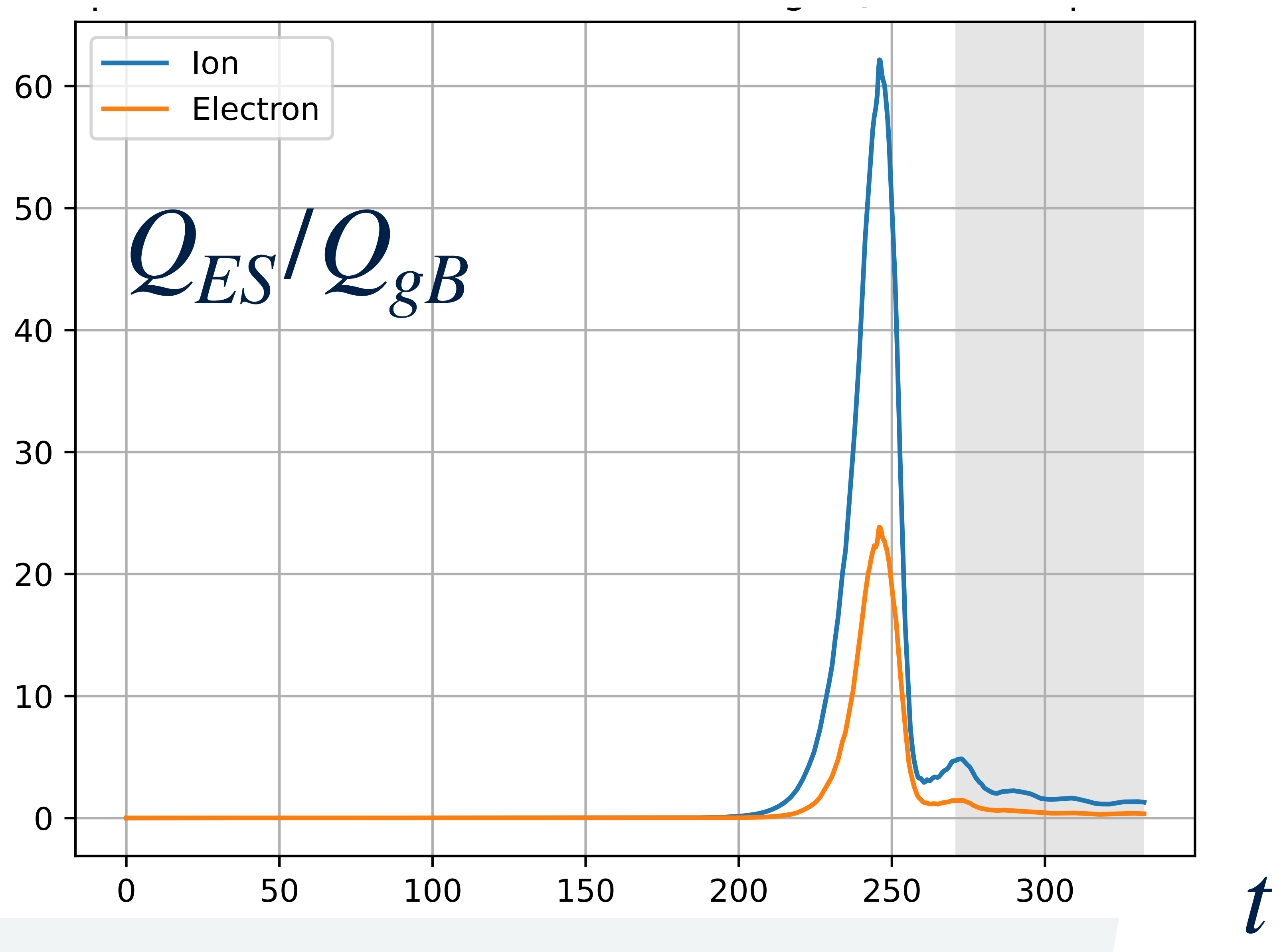
Numerical pulse 314, ASTRA RUN102 at time=450ms.

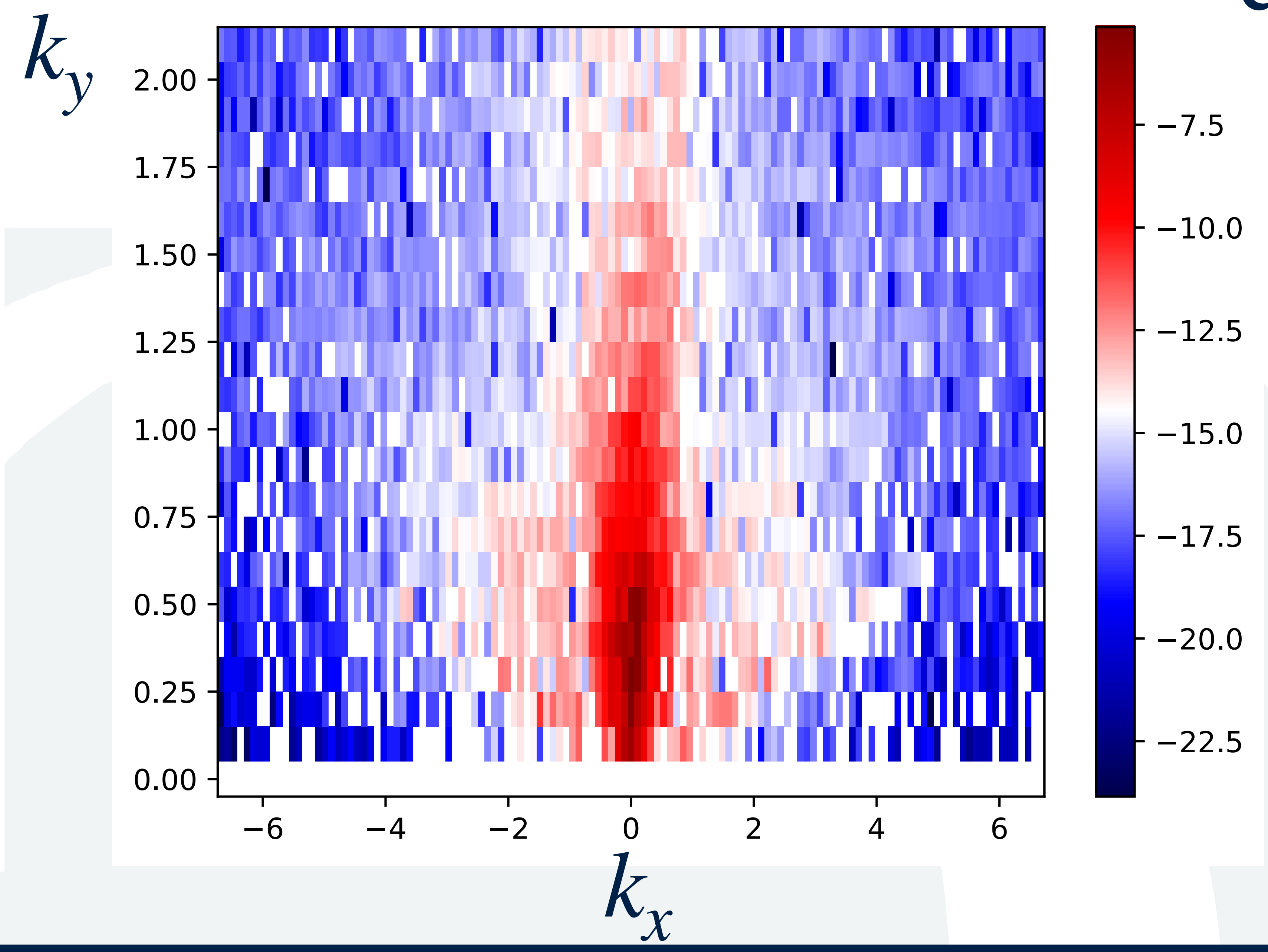


Heat flux time traces

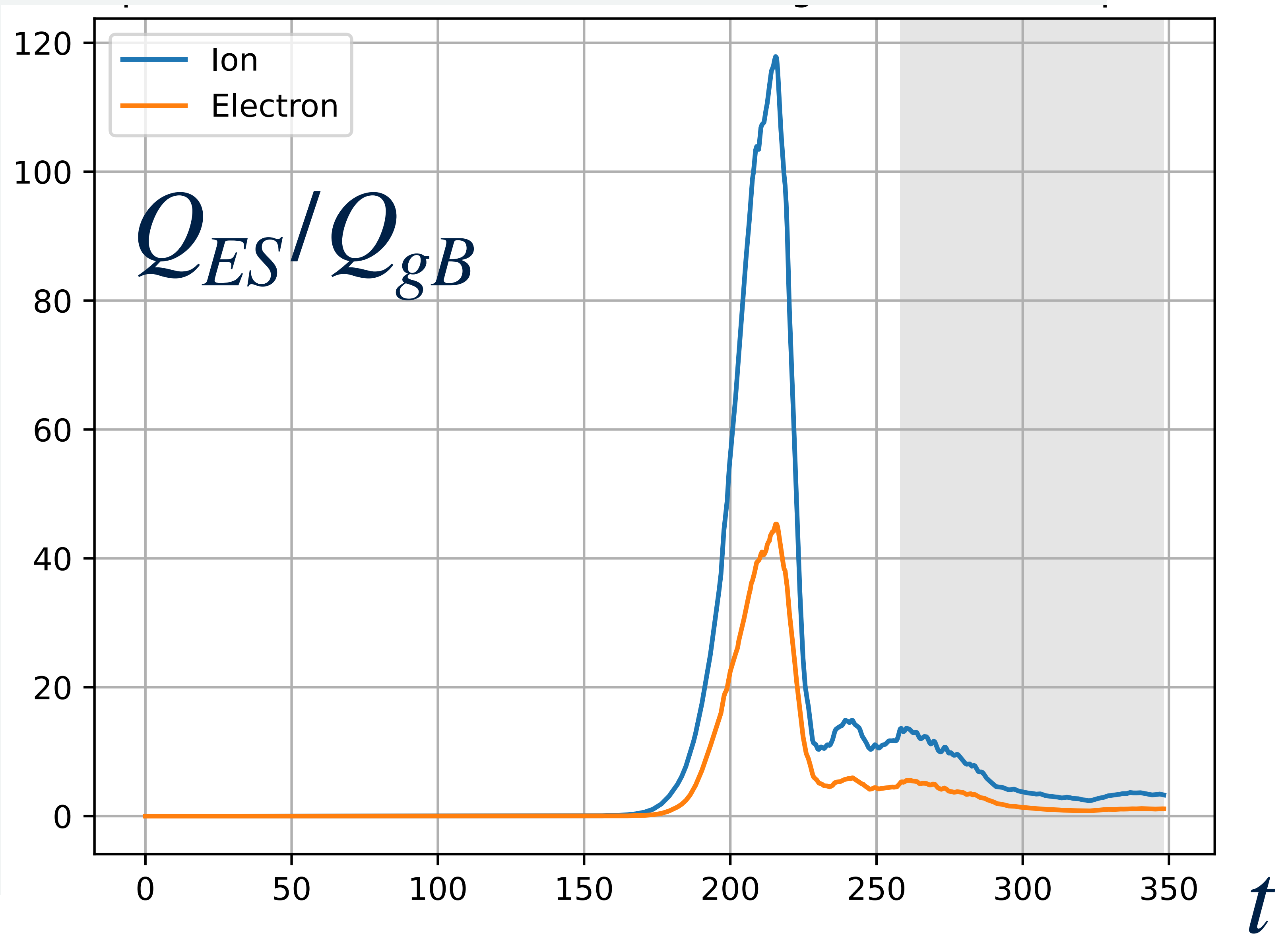
Heat flux spectra

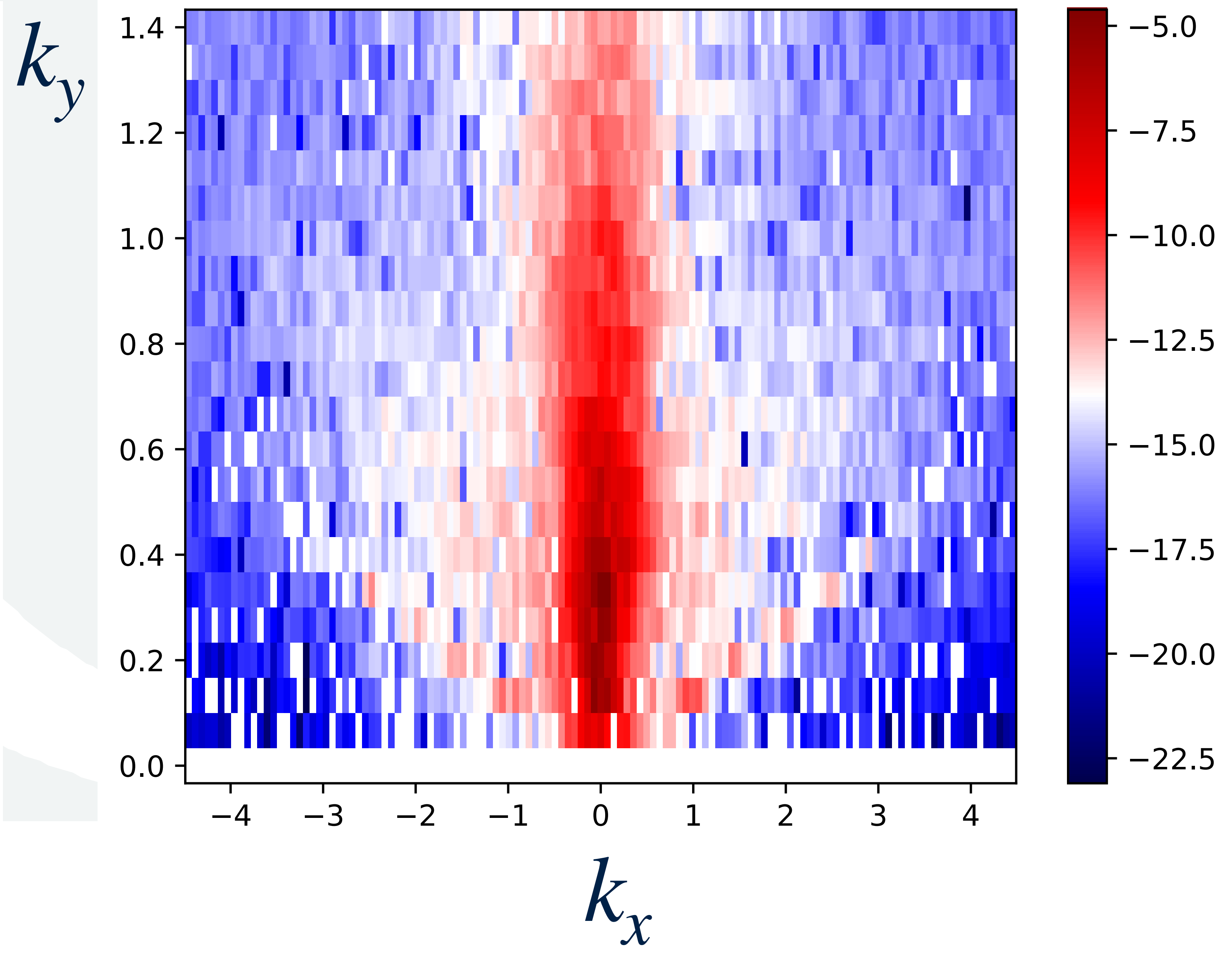
consistent $B_{\parallel}$ and β'





$q = 1.6$

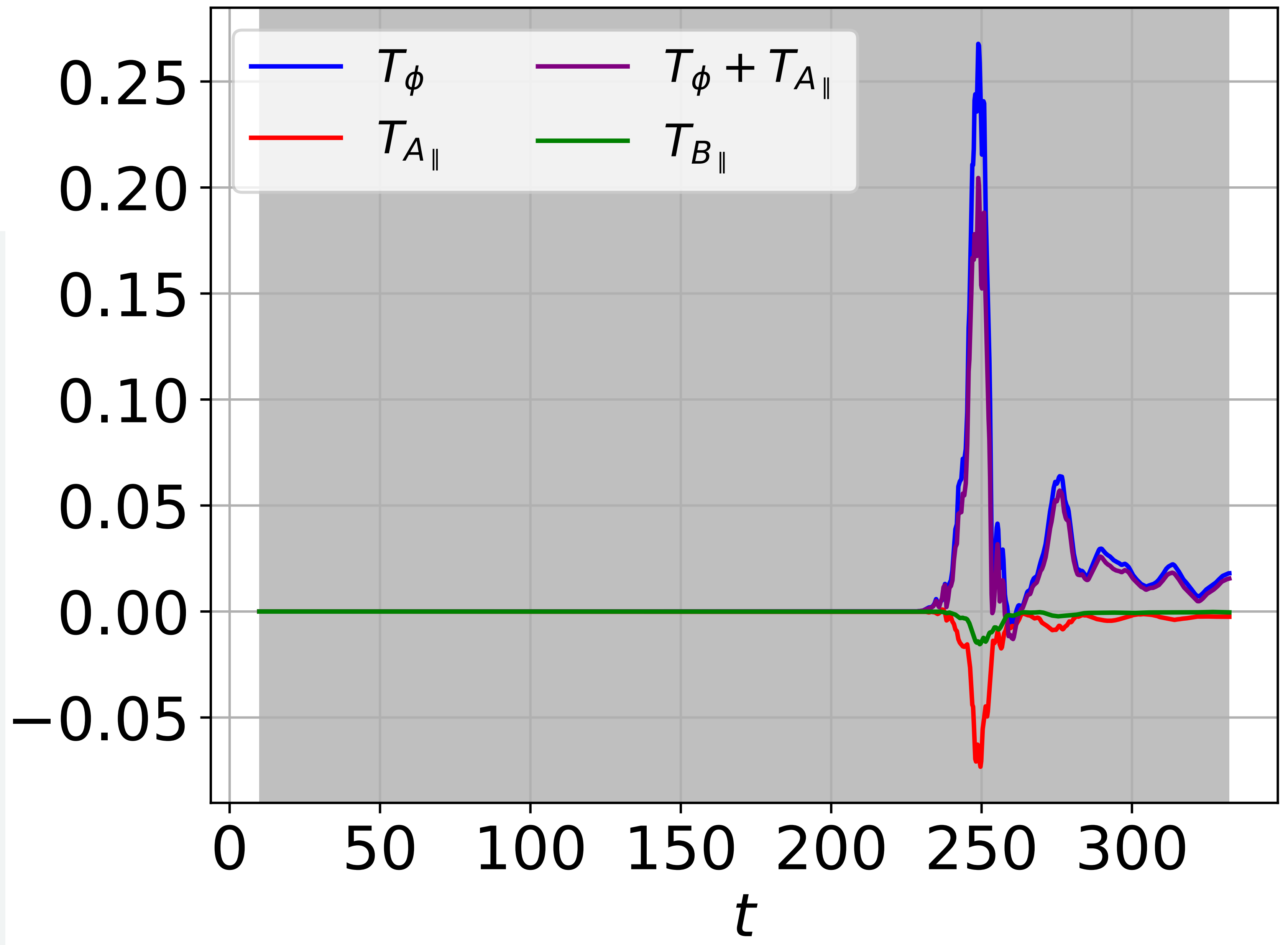


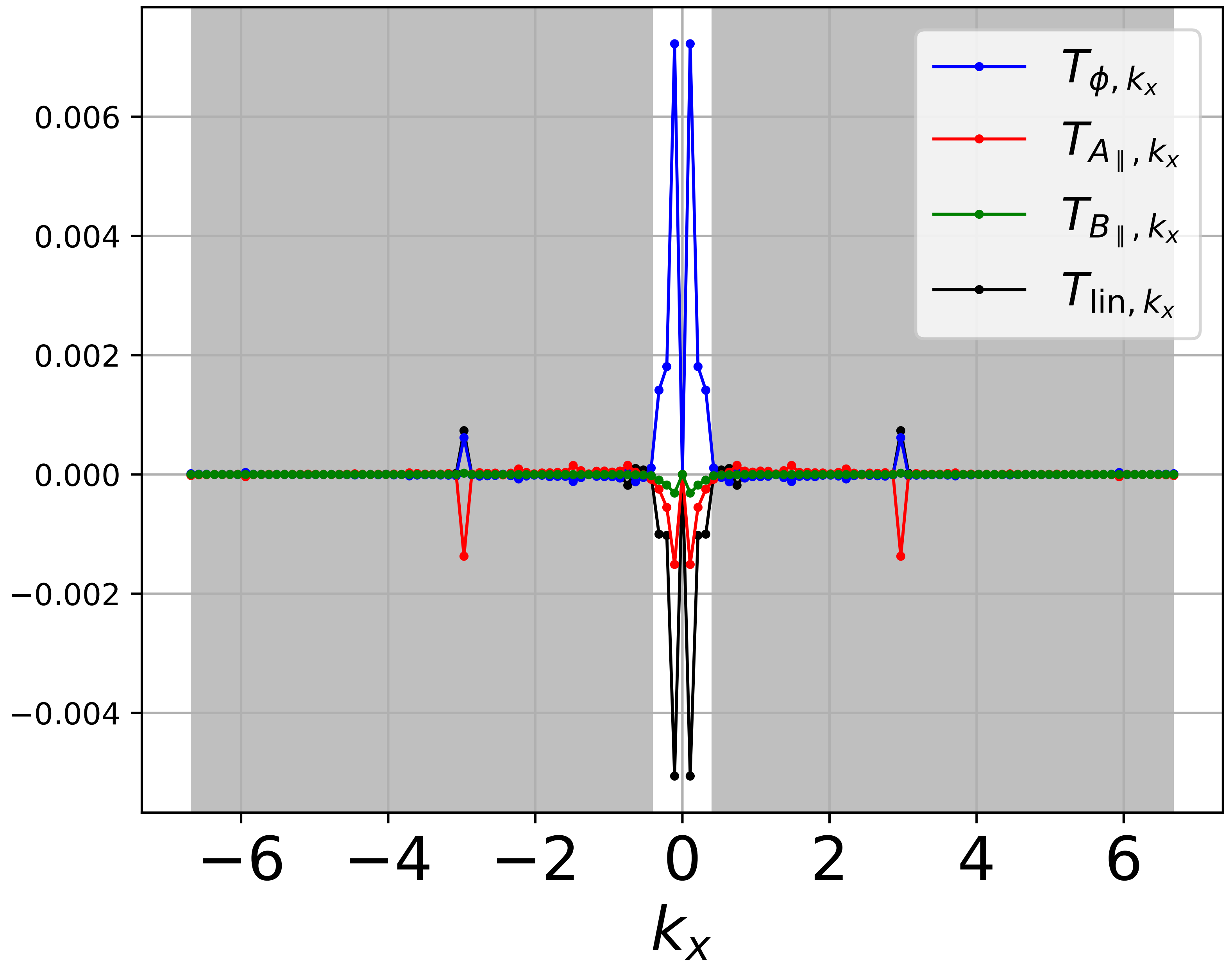


$q = 2.0$

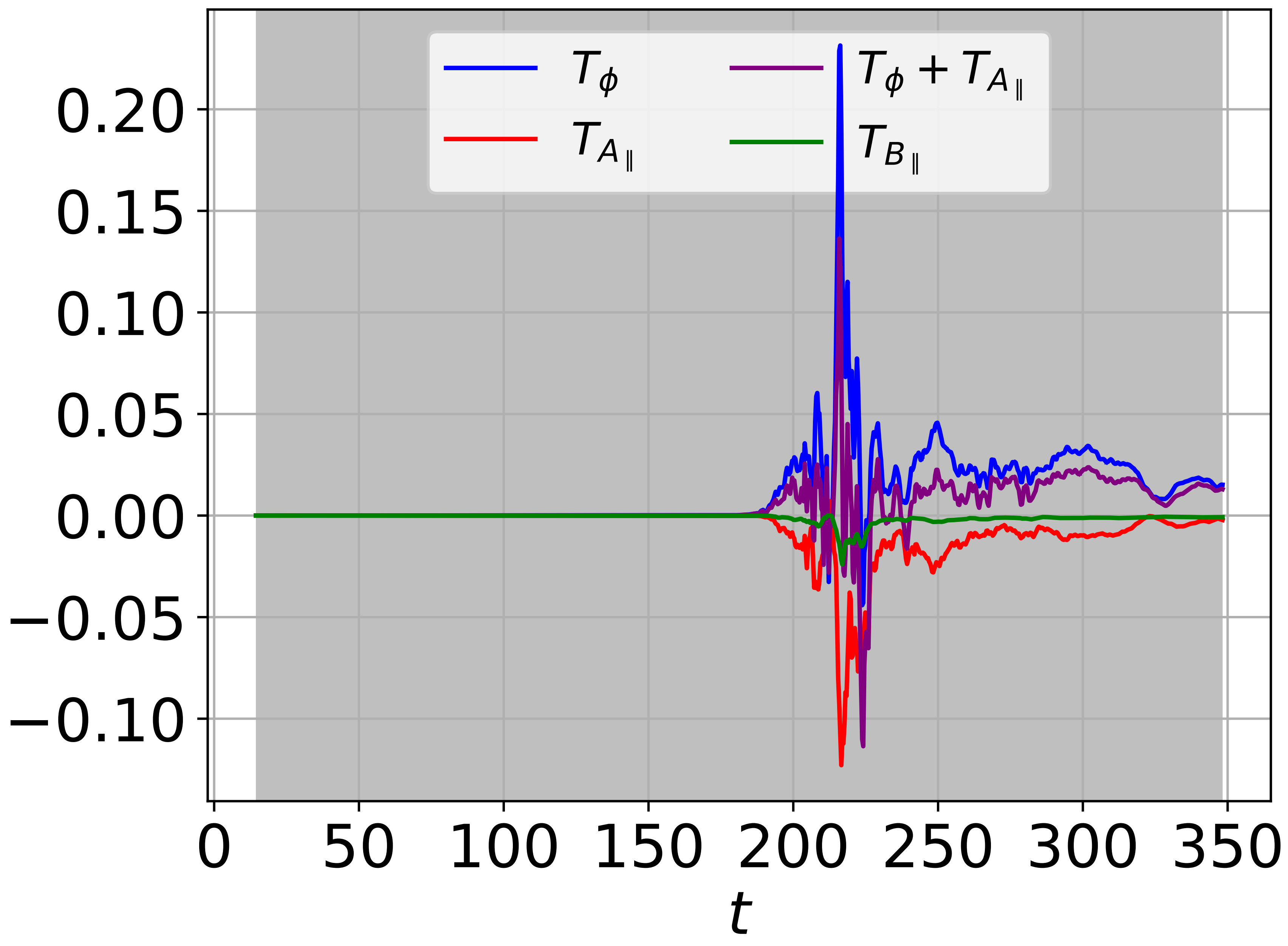
Large scale transfers

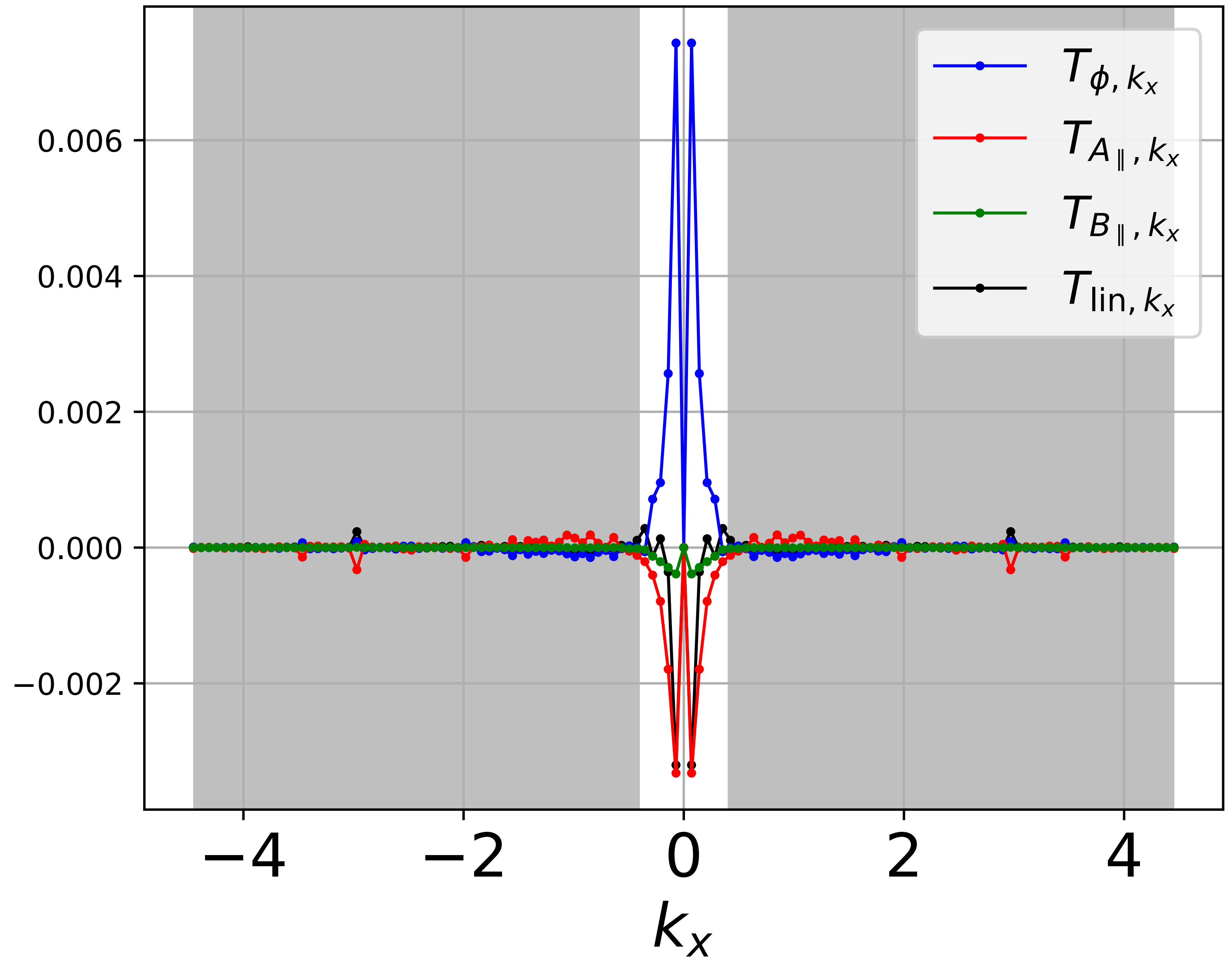
Transfer spectra





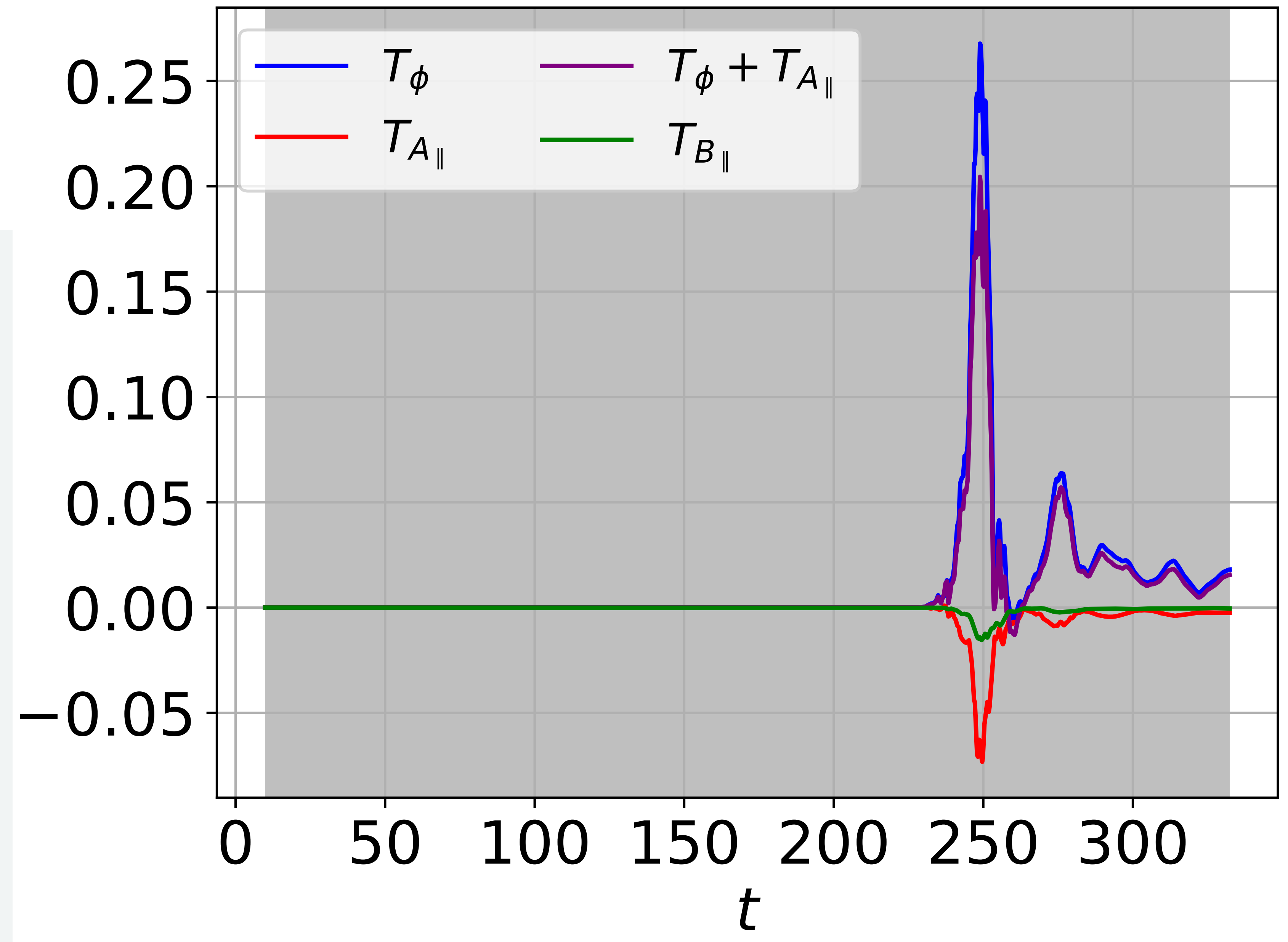
$q = 1.6$

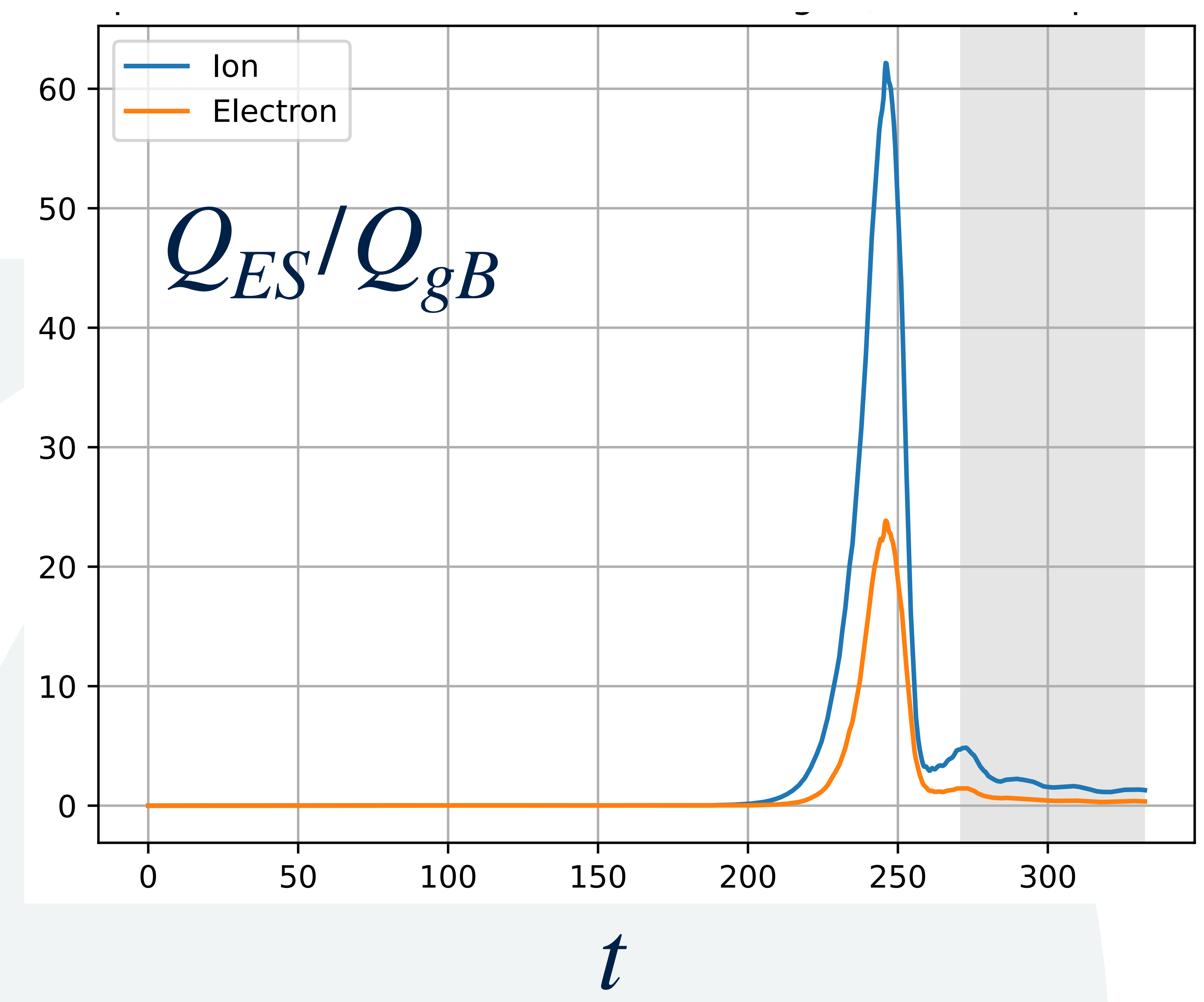




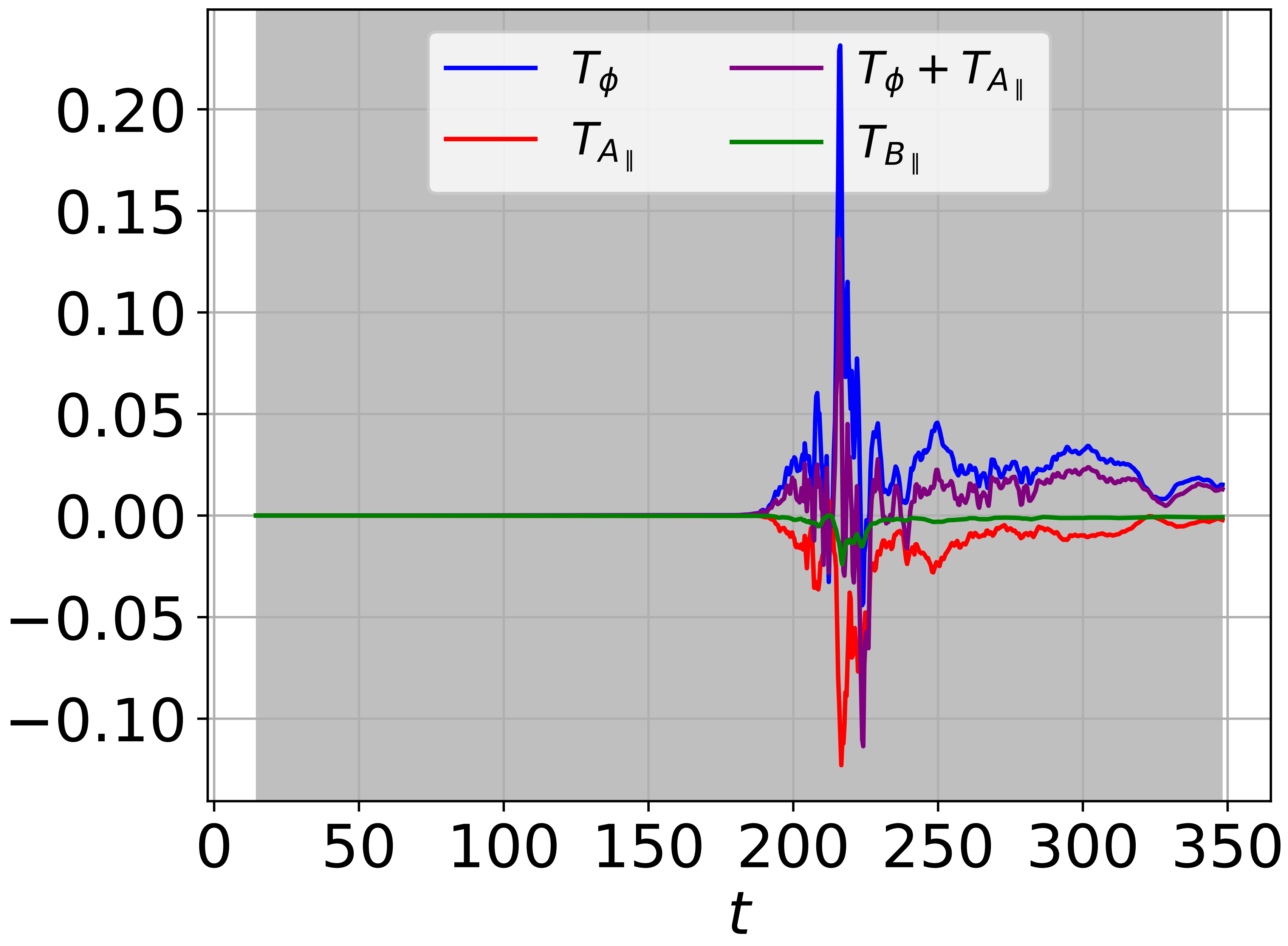
$q = 2.0$

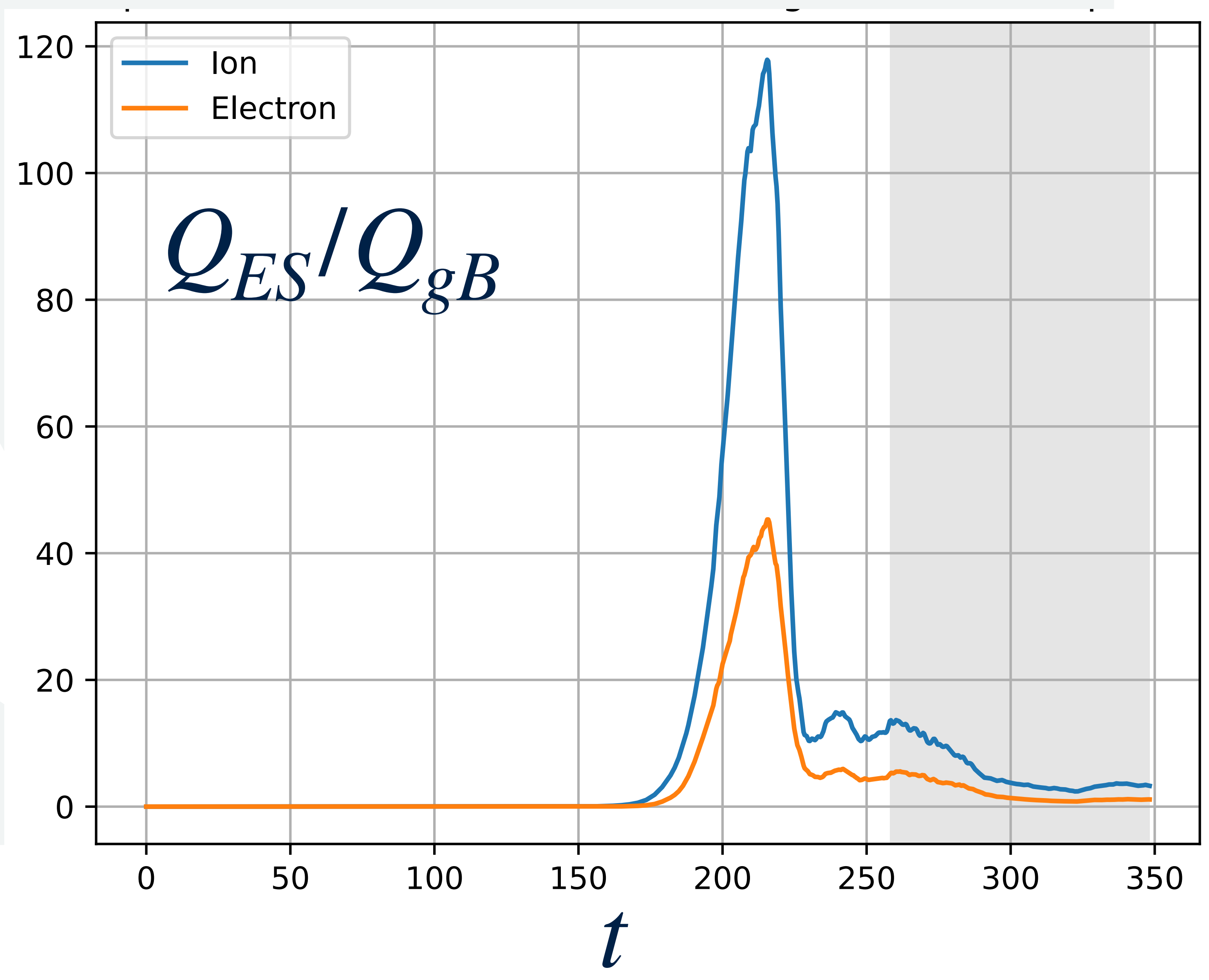
Large scale transfers





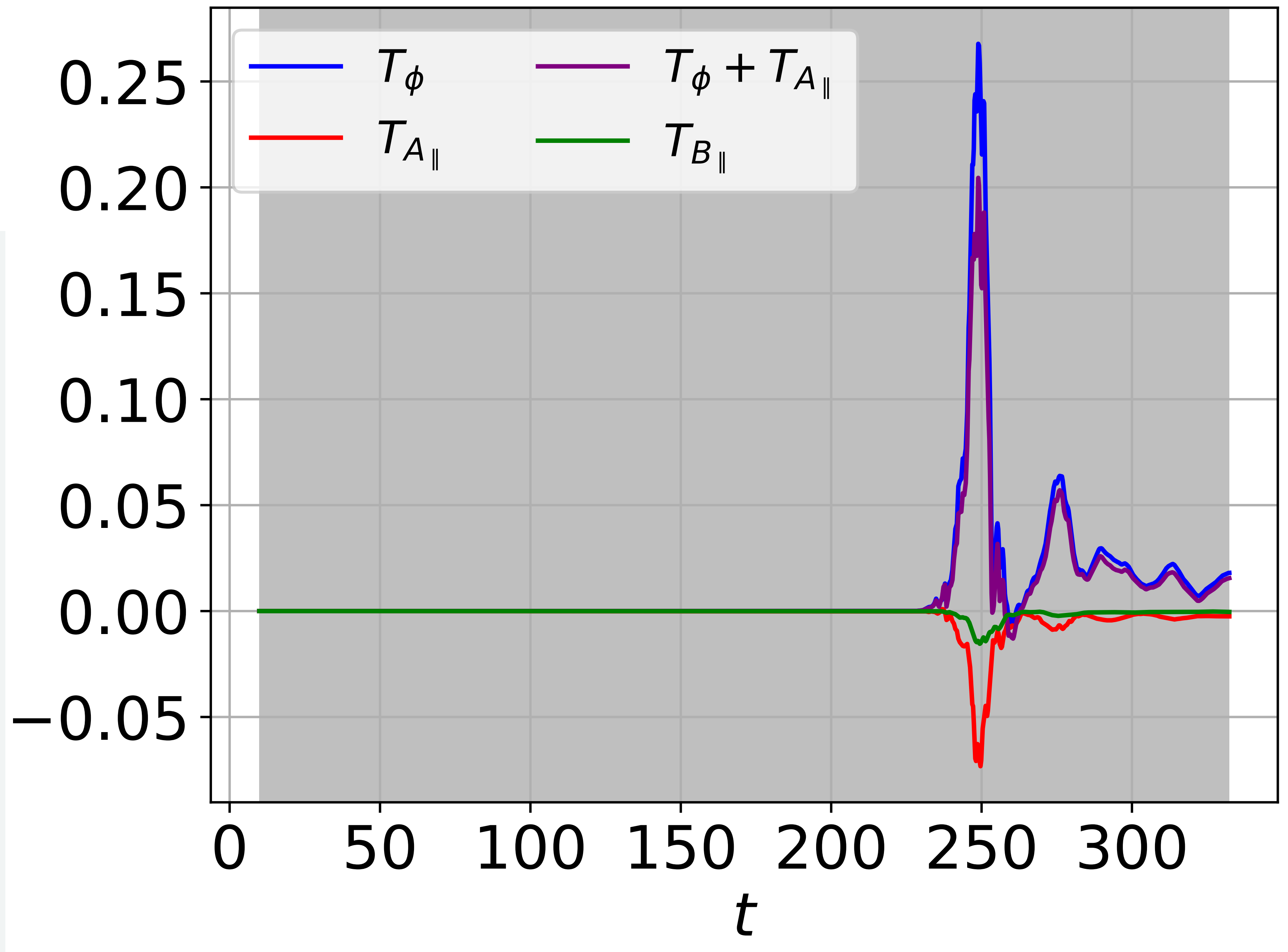
$q = 1.6$

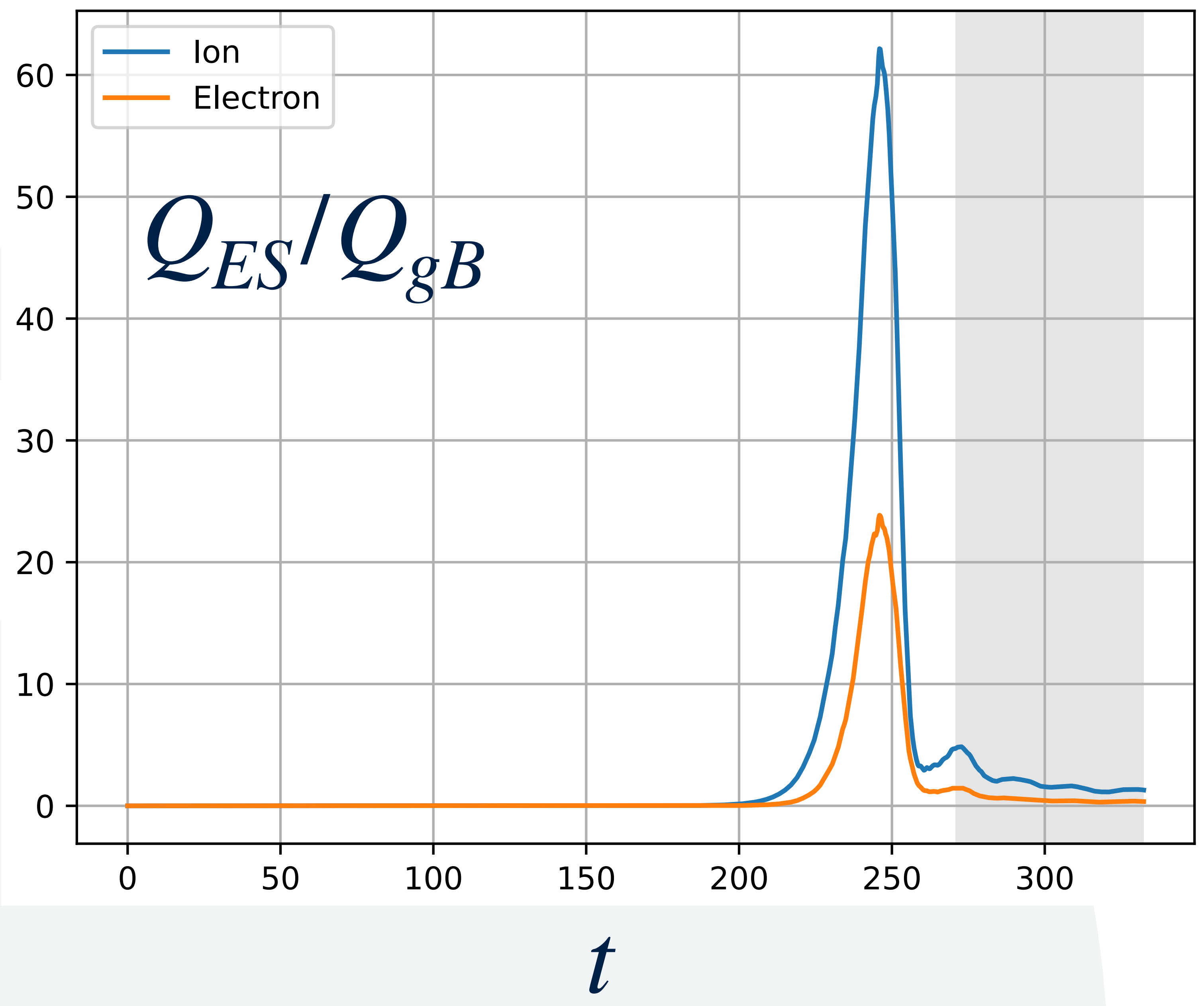




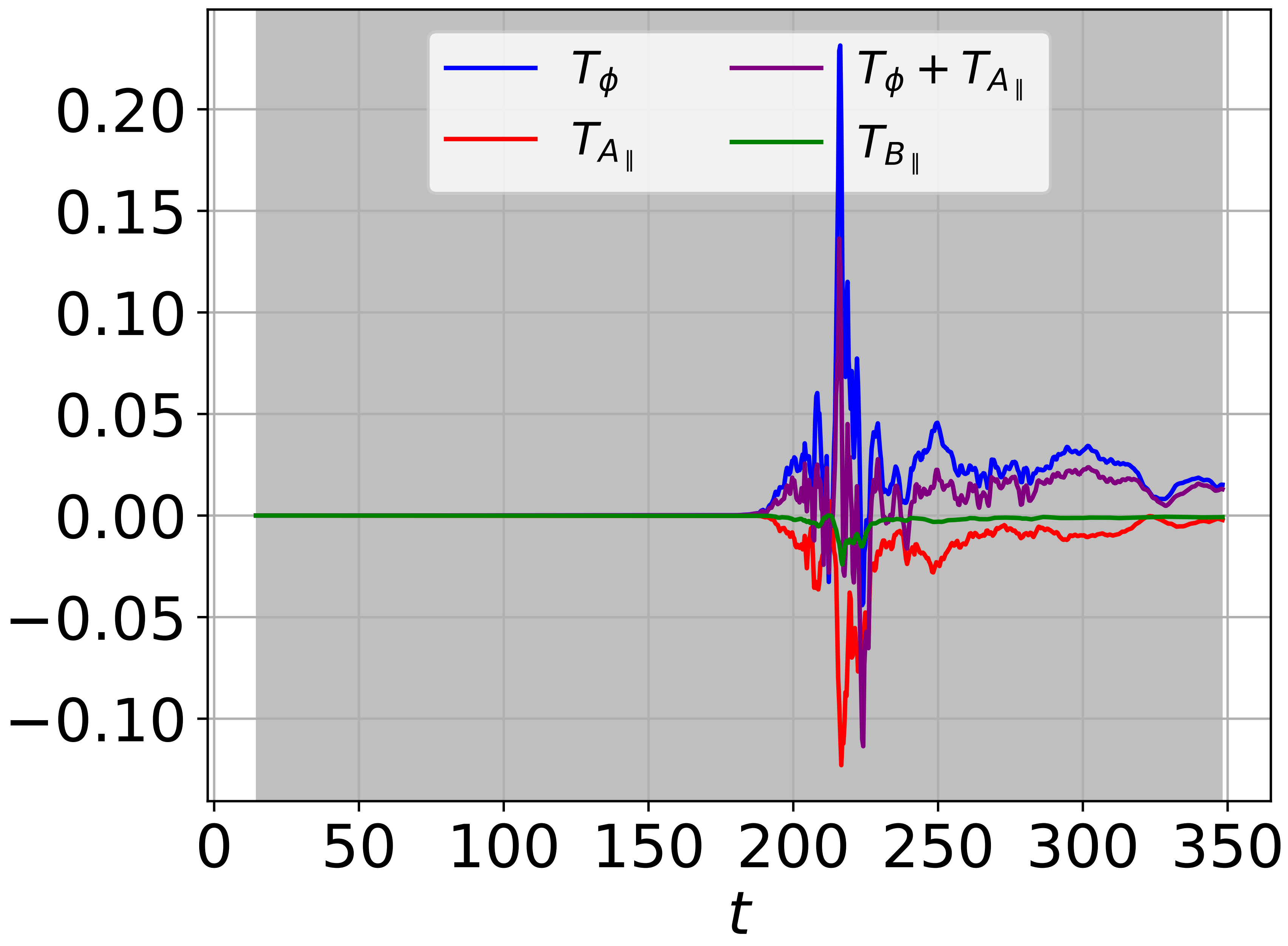
$q = 2.0$

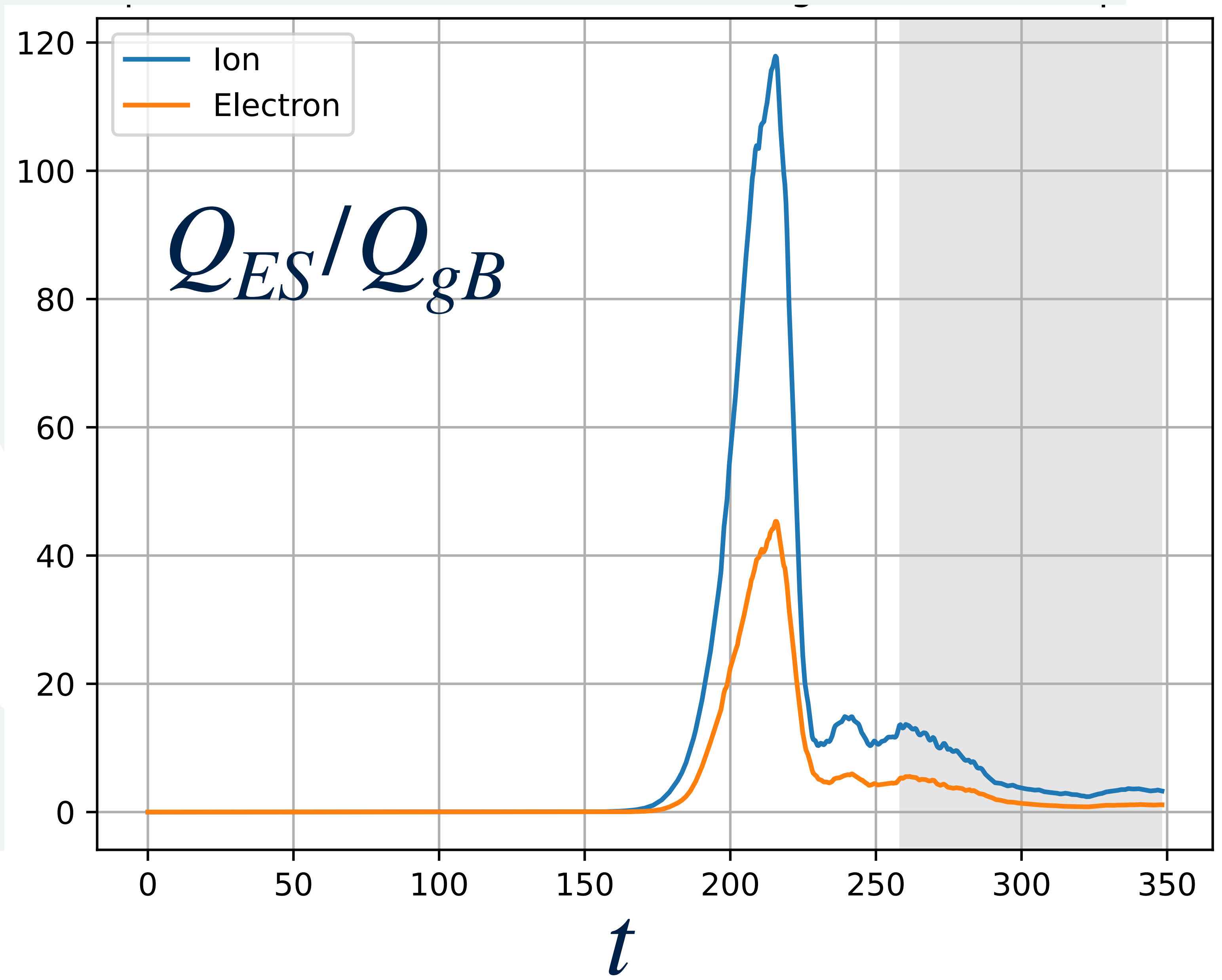
Large scale transfers





$q = 1.6$



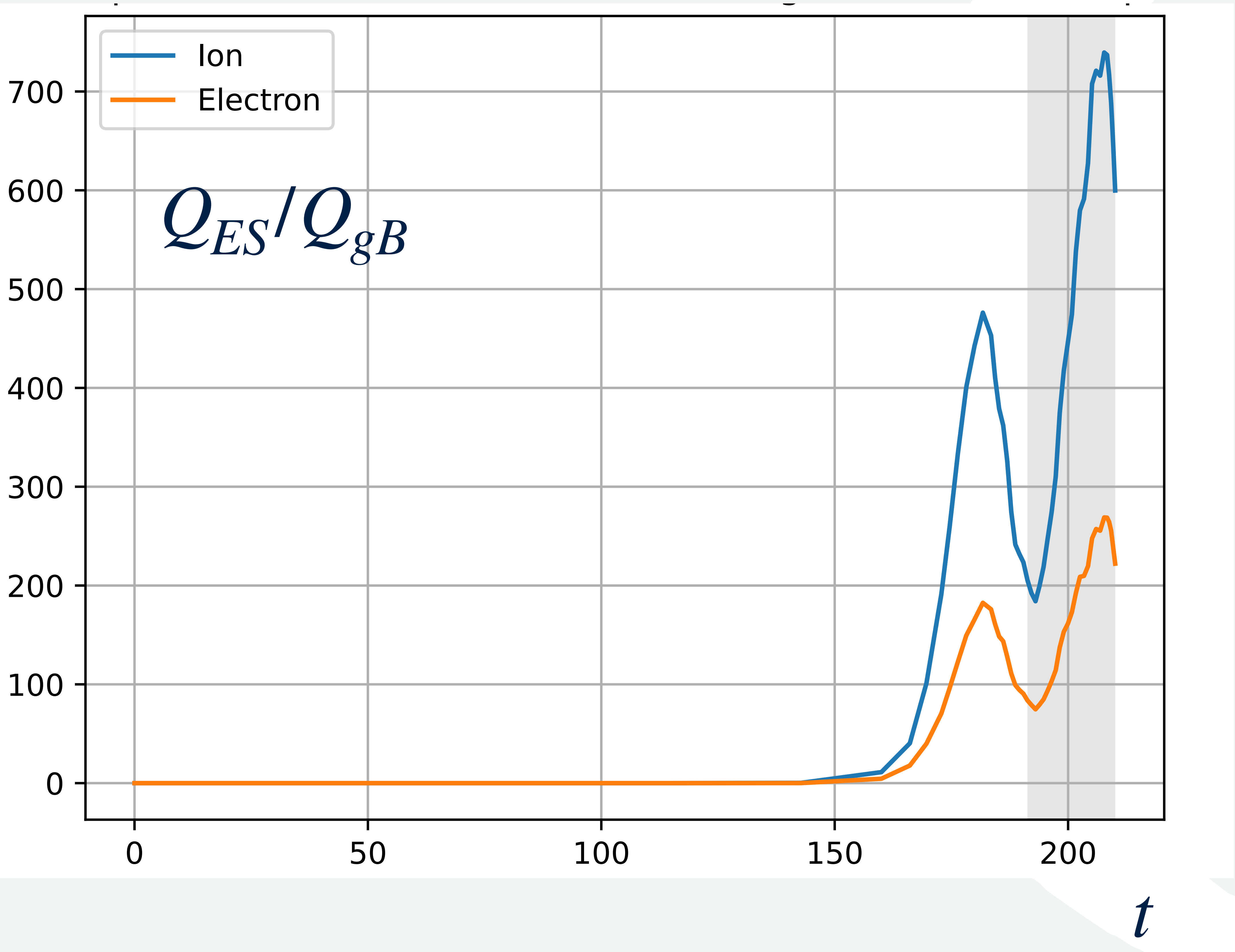


$q = 2.0$

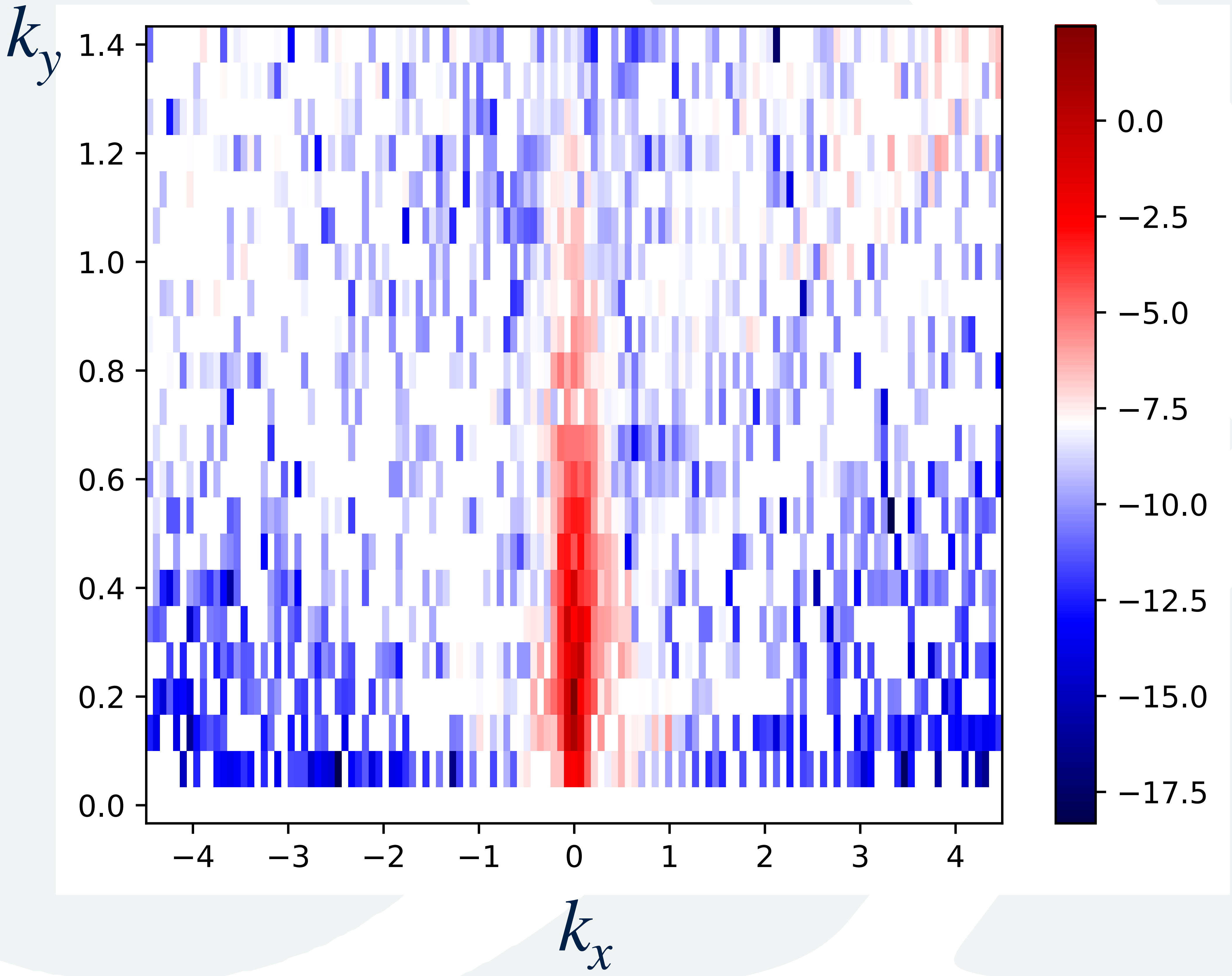
Guess what happens at larger q?

$q = 2.4$

Heat flux time trace

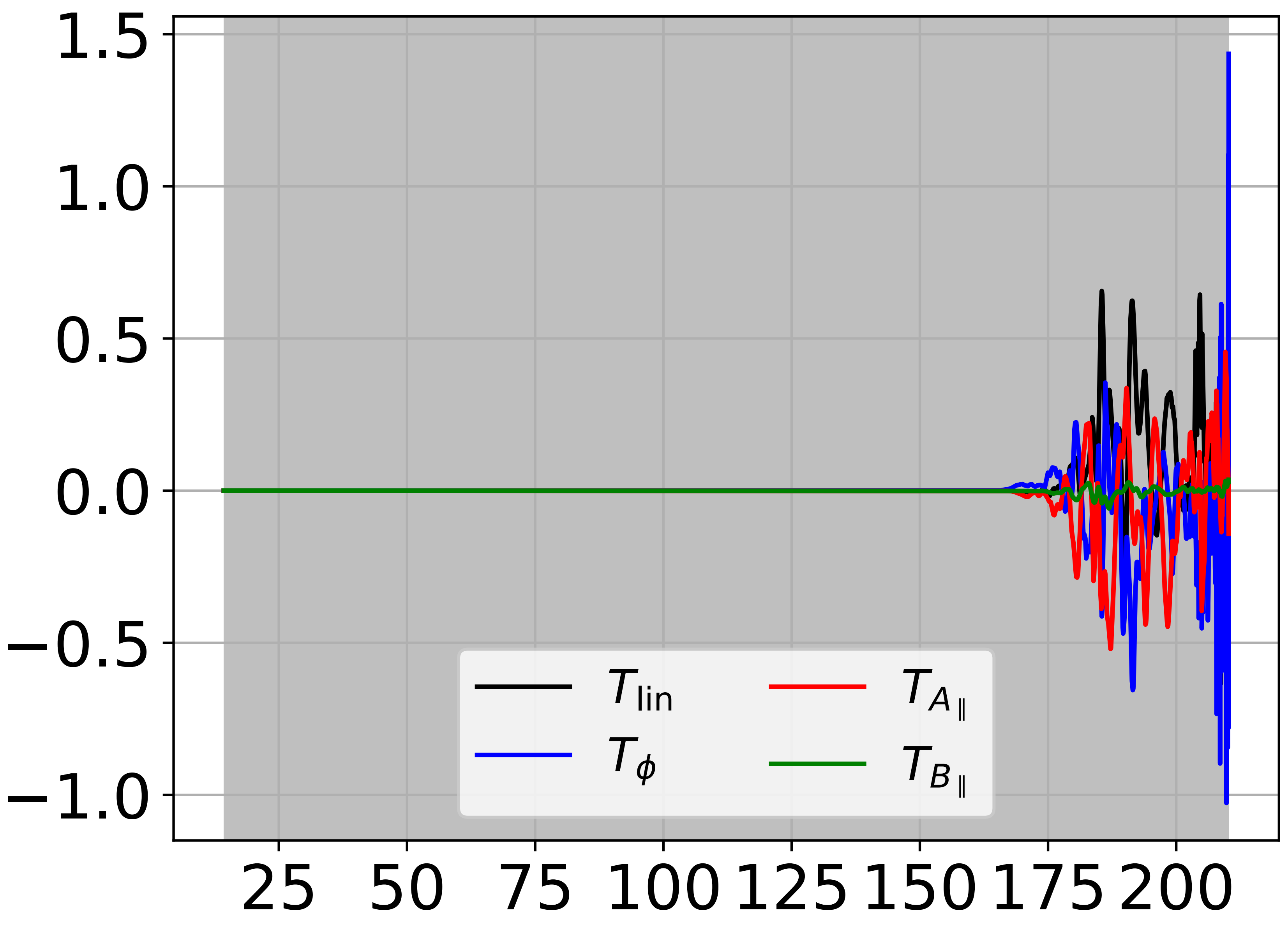


Heat flux spectrum

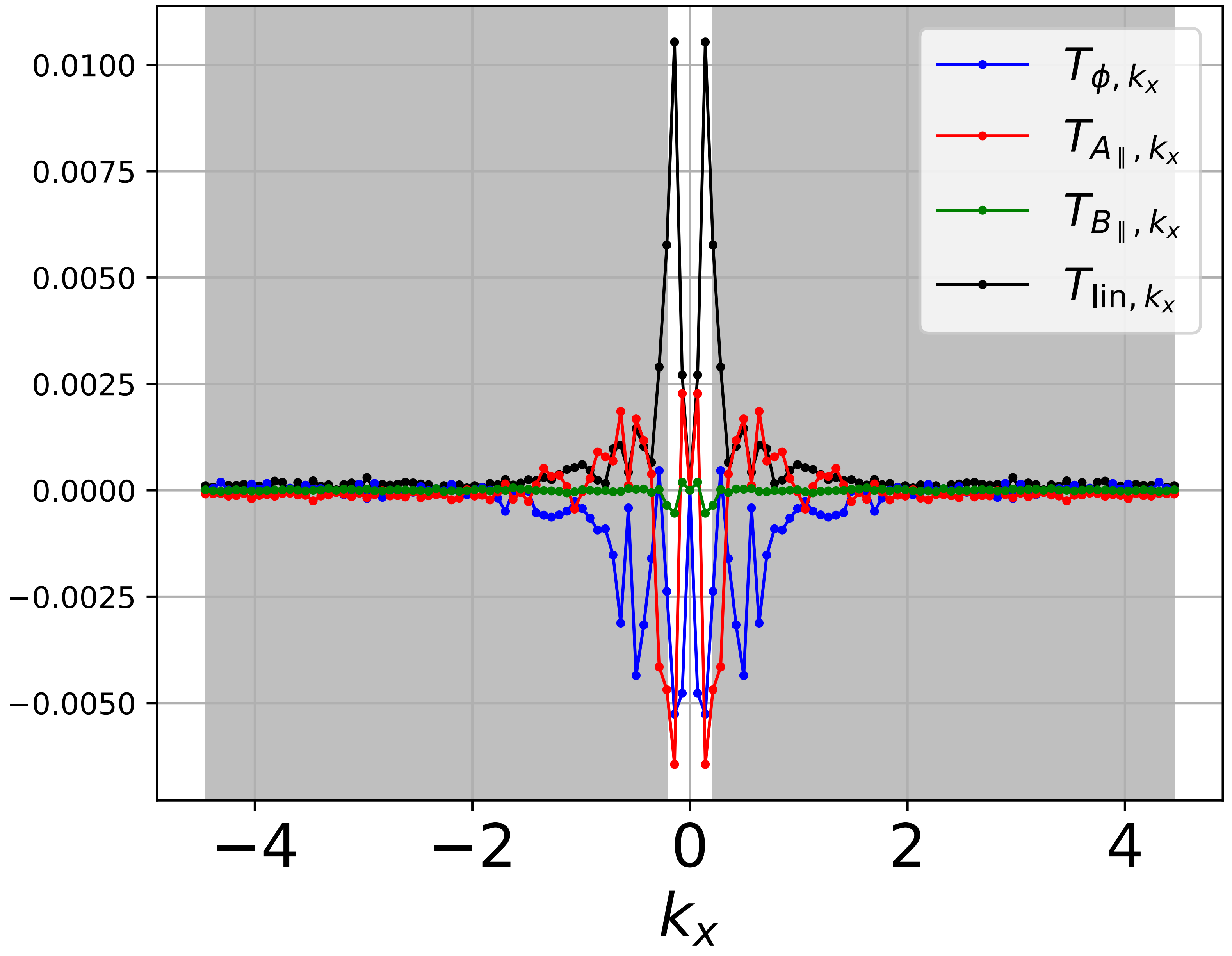


$q = 2.4$

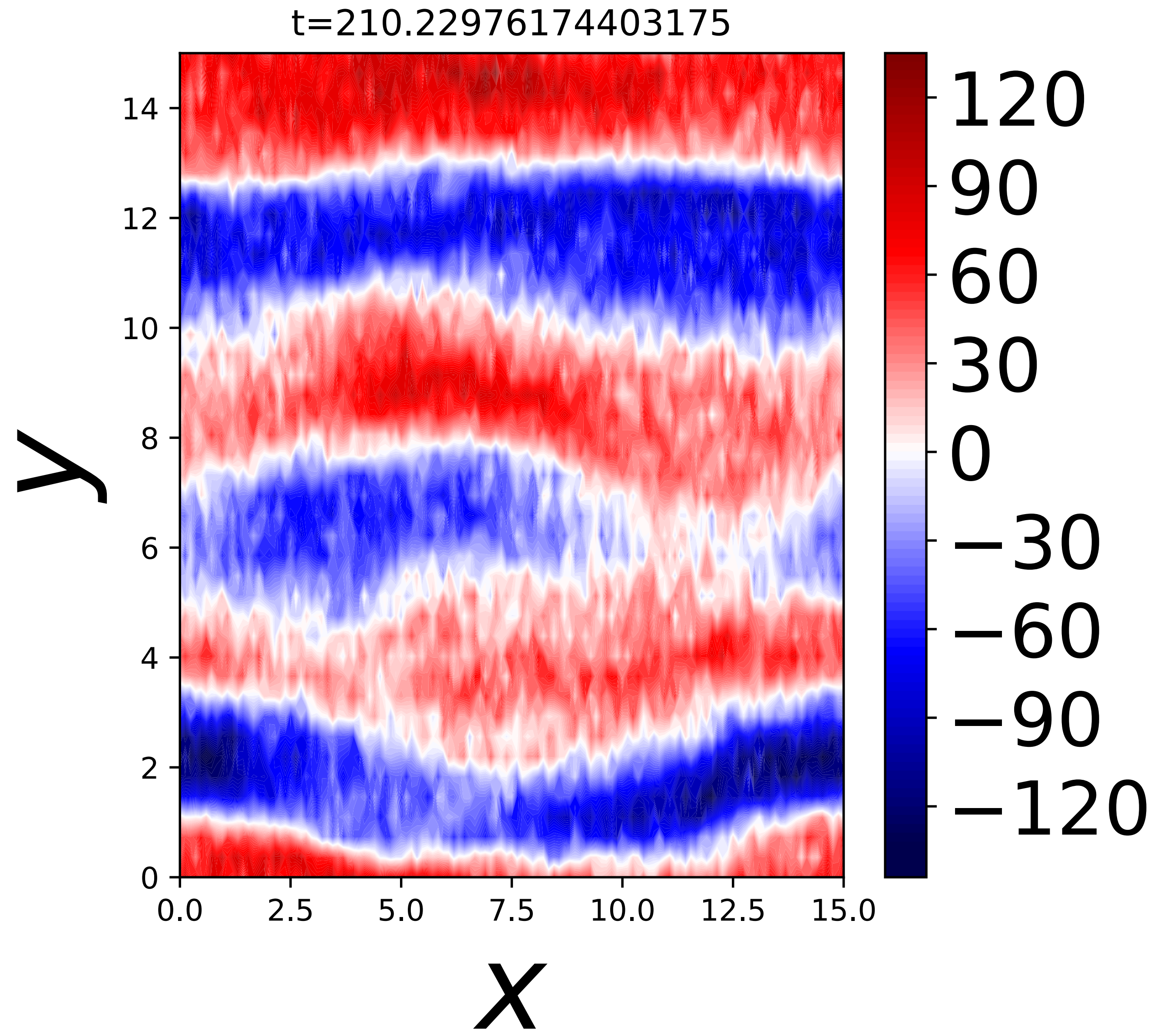
Large scale transfers



Transfer spectrum



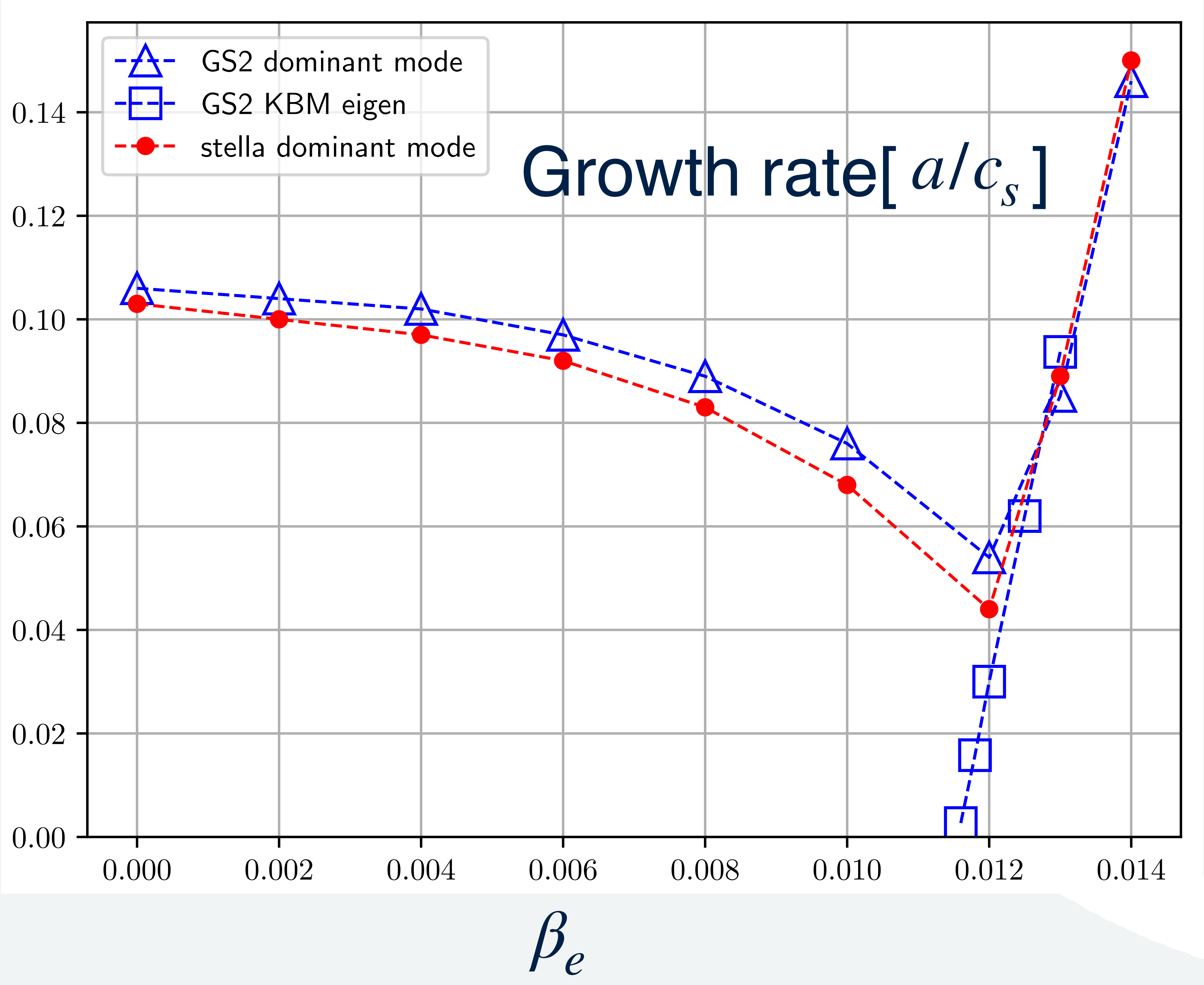
$\phi(z = 0)$

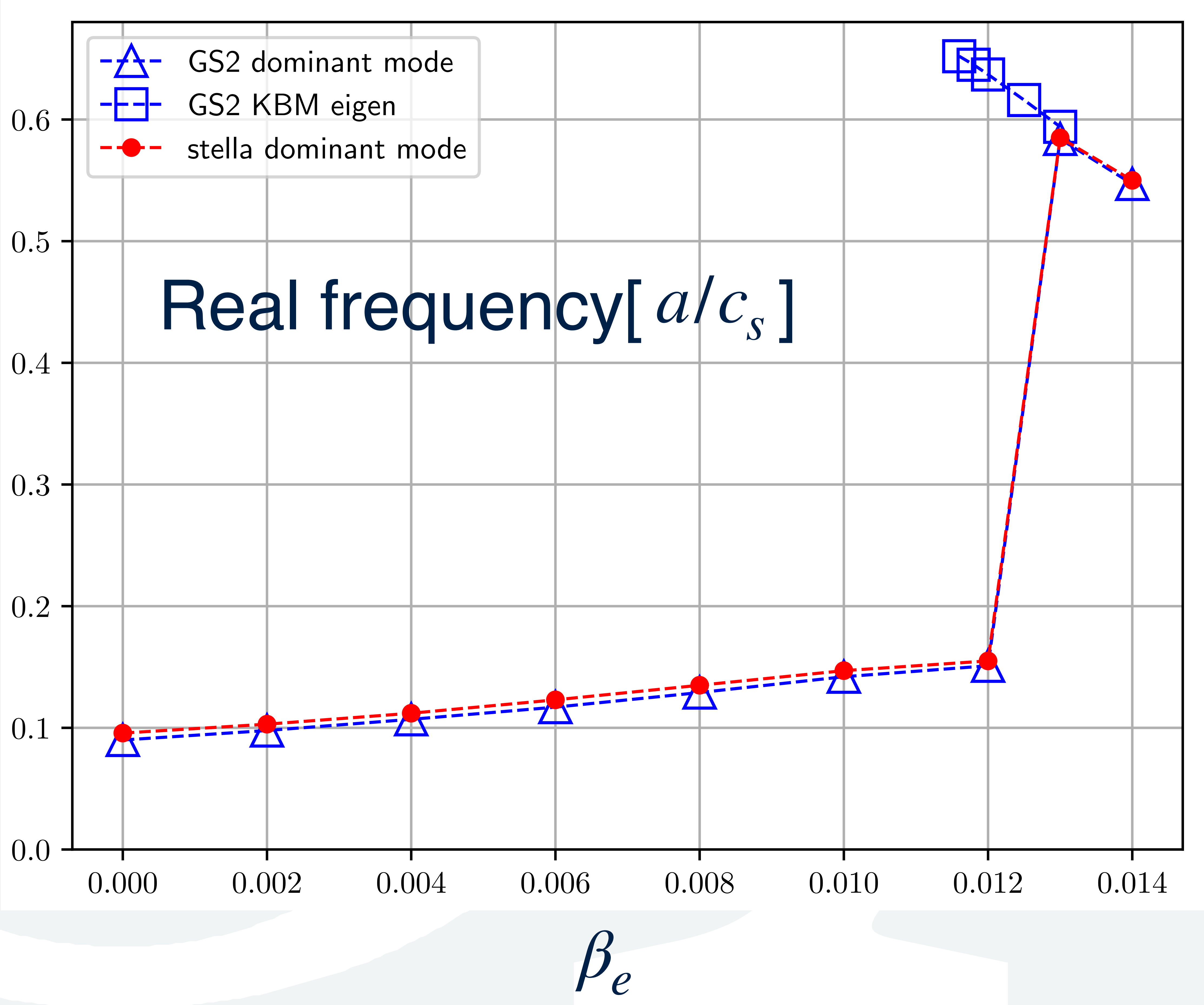


Benchmark

Linear benchmark (*stella* and *GS2*)

CBC, q = 1.4

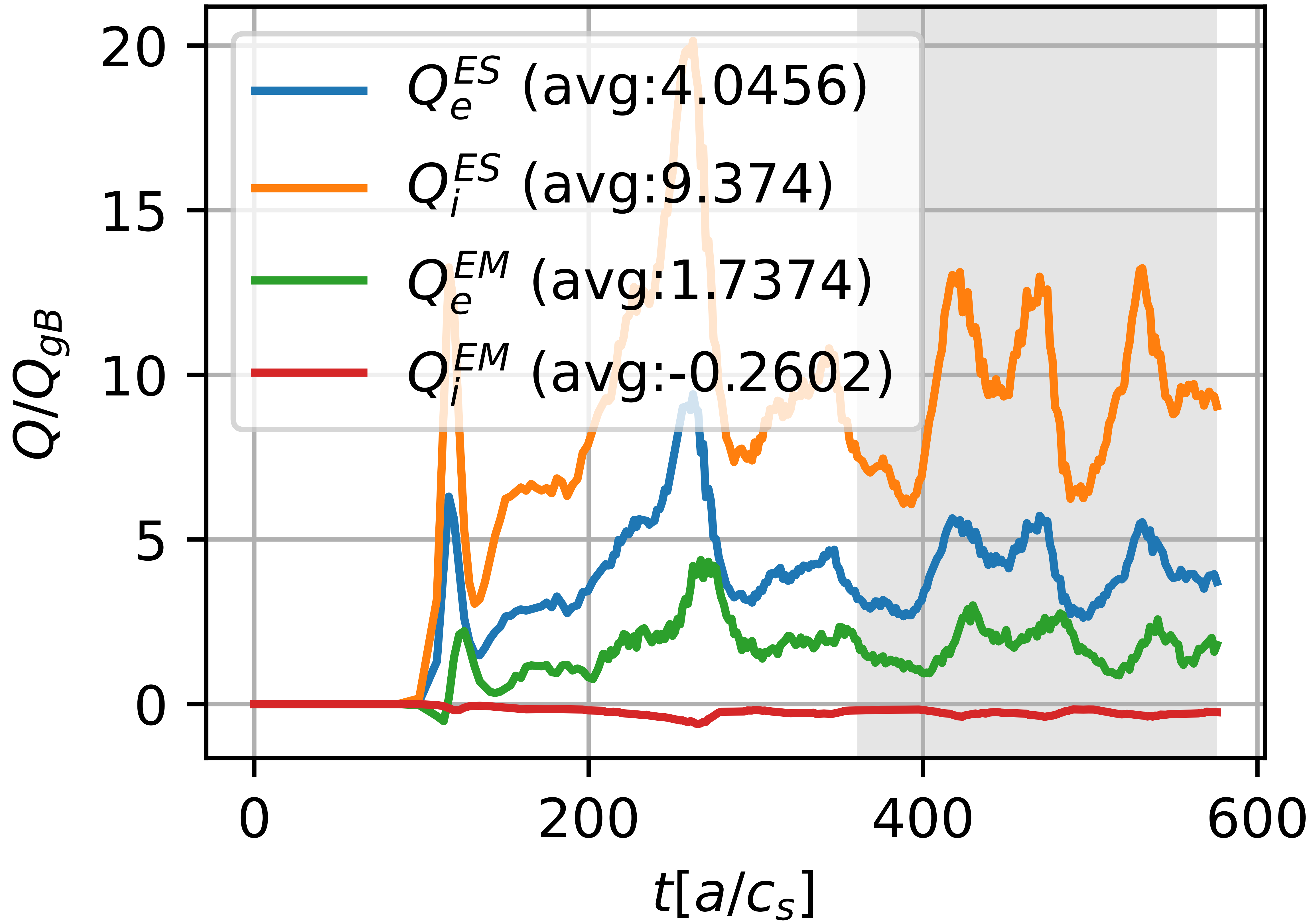




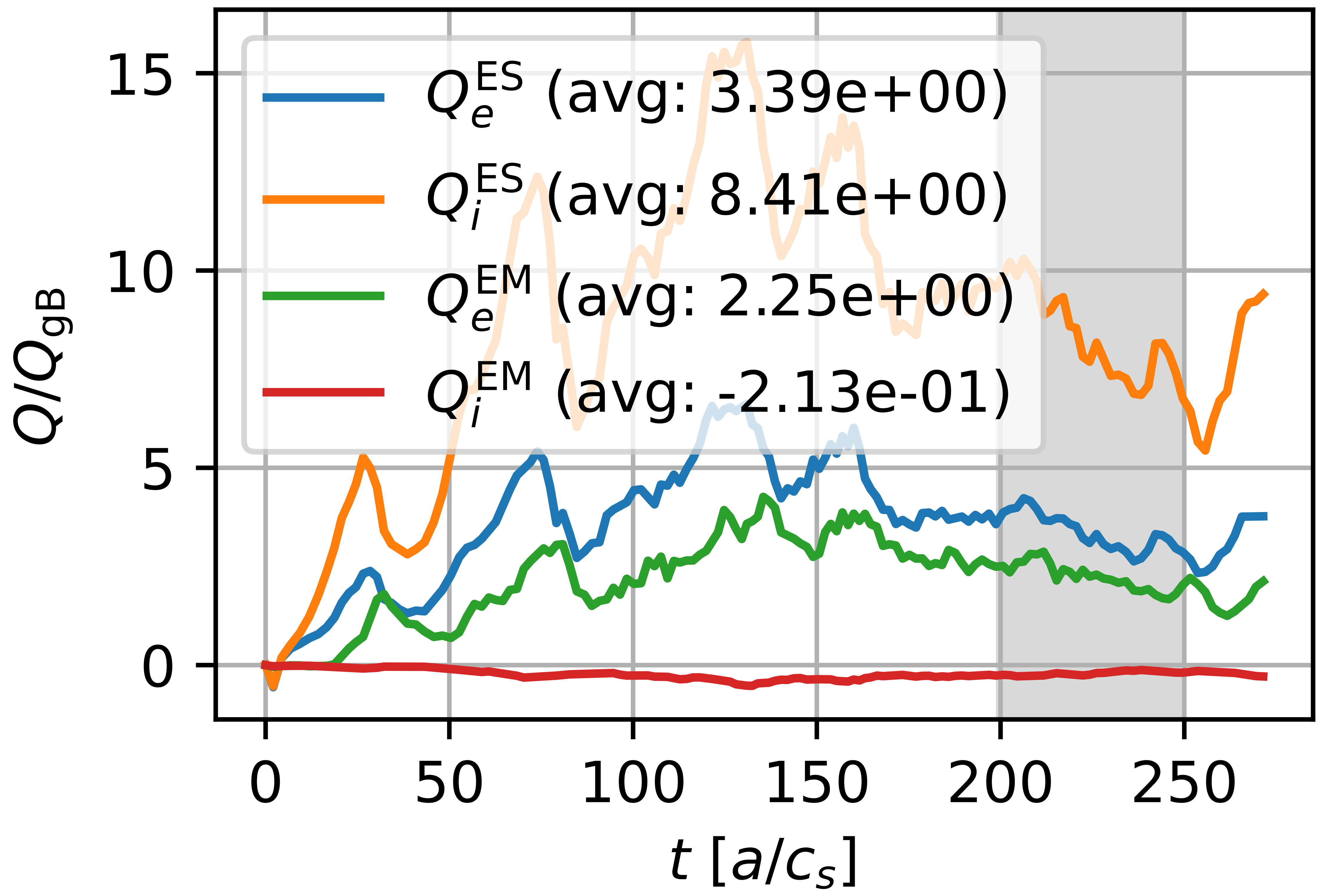
Nonlinear benchmark (*stella* and *GENE*)

CBC, q = 1.4, $\beta_e = 0.006$ consistent $B_{\|}$ and β'

stella



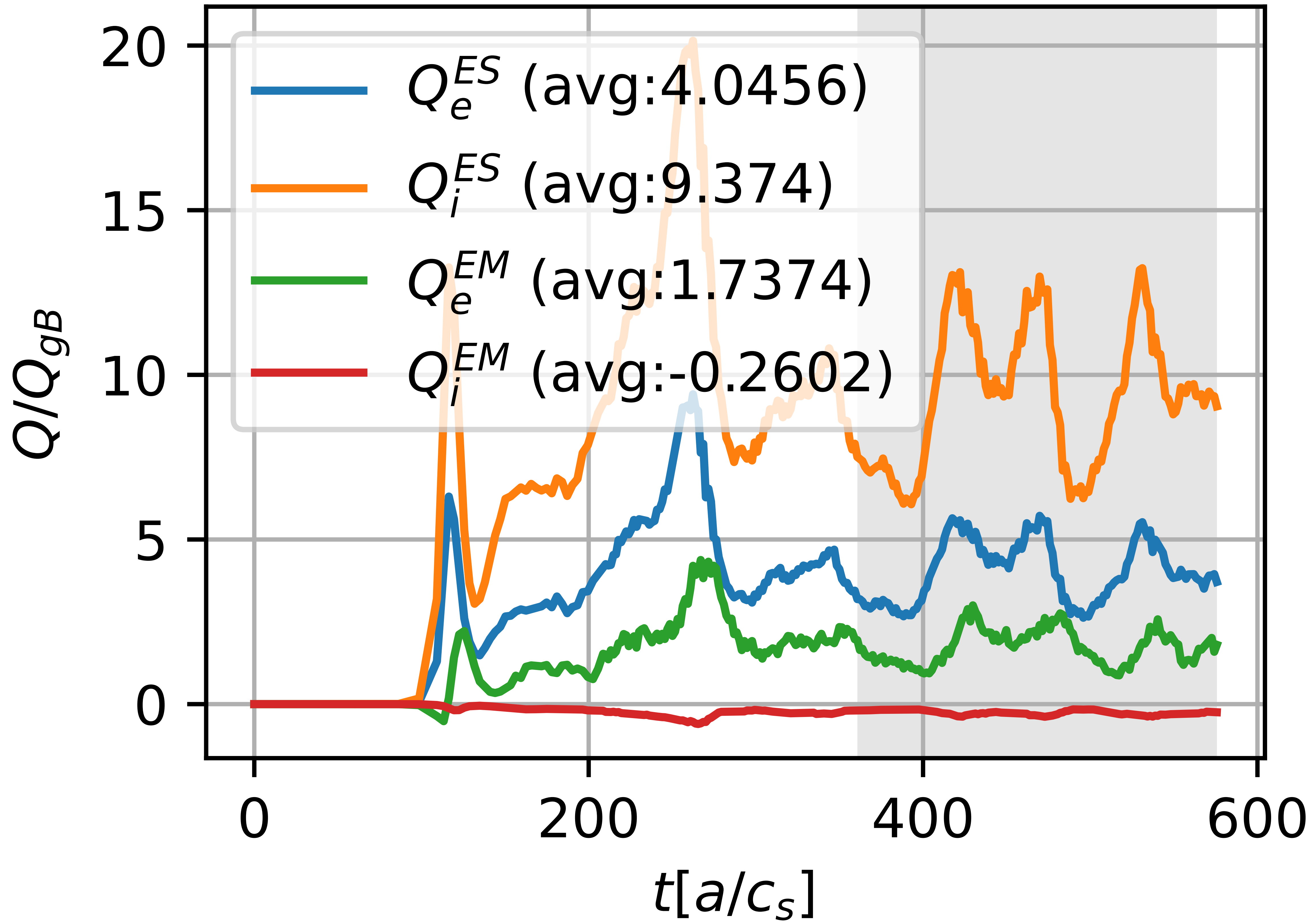
GENE D. Kennedy



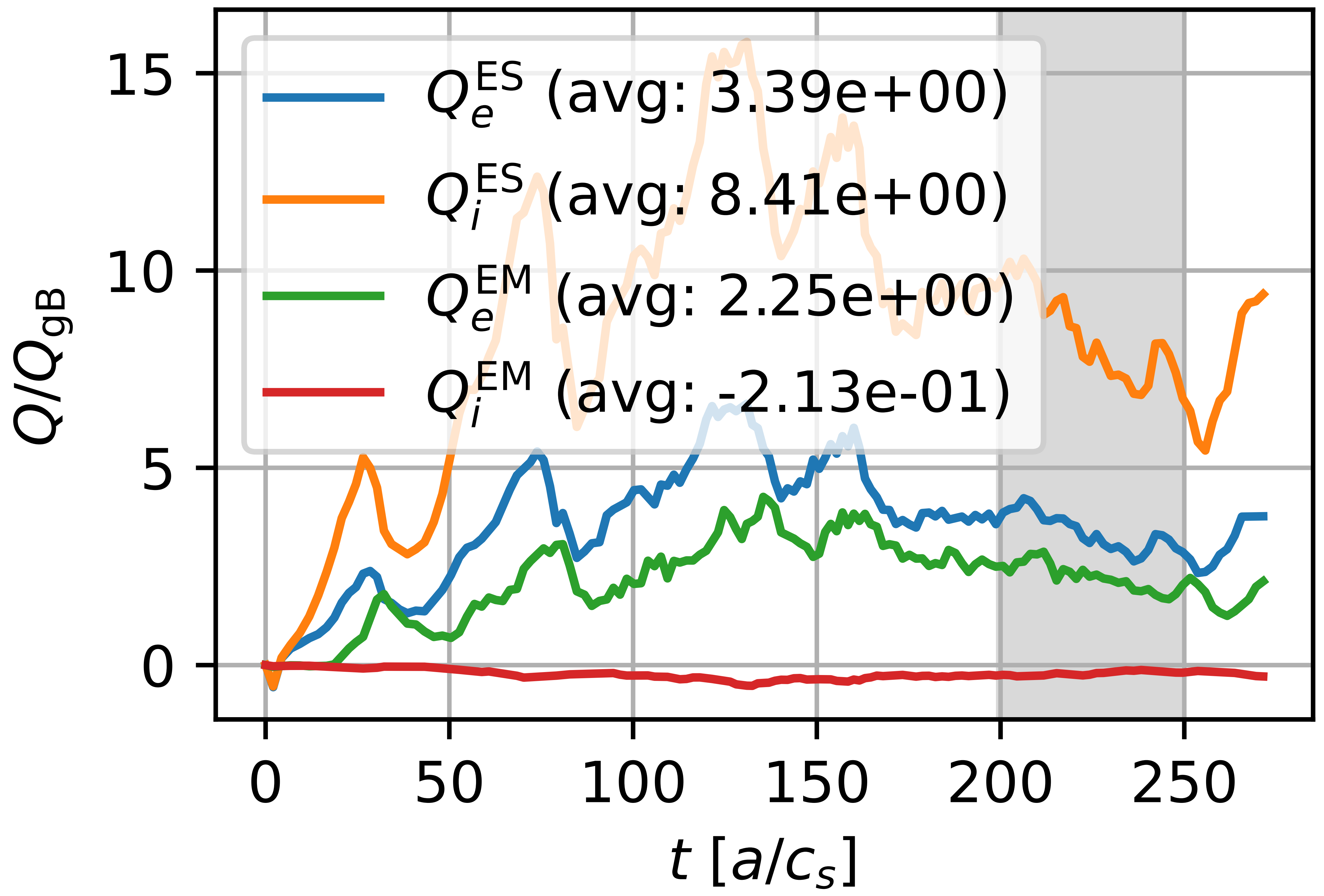
Nonlinear benchmark (*stella* and *GENE*)

CBC, q = 1.4, $\beta_e = 0.006$ consistent $B_{\parallel}$ and β'

stella



GENE D. Kennedy



Decent agreement, perhaps running them for longer can improve it more

Conclusion

1. $q^2\beta_e$ can be regarded as an effective β parameter for both linear instabilities and nonlinear runaway transition.

2. Evidence from CBC and ST40 cases suggests that the observed runaway transition is due to the cancellation between nonlinear stresses — Reynolds and Maxwell stresses (related work: *Rath & Peeters 2022).*

3. Local flux-tube *Stella* is working electromagnetically.

Thanks for your attention!